

University of Mumbai

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विद्याविषयक प्राधिकरणे
सभा आणि सेवा विभाग (ए.ए.एम.एस)
रूम नं. १२८ एम.जी.रोड, फोर्ट,
मुंबई - ४०० ०३२
टेलिफोन नं - ०२२ - ६८३२००३३

(नॅक पुनर्मूल्यांकनाद्वारे ३.६५ (सी.जी.पी.ए.) सह अ++ श्रेणी
विद्यापीठ अनुदान आयोगाद्वारे श्रेणी १ विद्यापीठ दर्जा)

क्र.वि.प्रा.स.से./आयसीडी/२०२५-२६/३७

दिनांक : २७ मे, २०२५

परिपत्रक:-

सर्व प्राचार्य/संचालक, संलग्नित महाविद्यालये/संस्था, विद्यापीठ शैक्षणिक विभागांचे संचालक/ विभाग प्रमुख यांना कळविण्यात येते की, राष्ट्रीय शैक्षणिक धोरण २०२० च्या अमलबजावणीच्या अनुषंगाने शैक्षणिक वर्ष २०२५-२६ पासून पदवी व पदव्युत्तर अभ्यासक्रम विद्यापरिषदेच्या दिनांक २८ मार्च २०२५ व २० मे, २०२५ च्या बैठकीमध्ये मंजूर झालेले सर्व अभ्यासक्रम मुंबई विद्यापीठाच्या www.mu.ac.in या संकेत स्थळावर NEP २०२० या टॅब वर उपलब्ध करण्यात आलेले आहेत.

मुंबई - ४०० ०३२
२७ मे, २०२५


(डॉ. प्रसाद कारंडे)
कुलसचिव

Copy forwarded for information and necessary action to :-	
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8	The Deputy Registrar, Executive Authorities Section (EA) eau120@fort.mu.ac.in He is requested to treat this as action taken report on the concerned resolution adopted by the Academic Council referred to the above circular.
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As Per NEP 2020

University of Mumbai



Syllabus for Major Vertical – 1 & 4 (Scheme – III)

Name of the Programme – B.A./B.Sc. (Mathematics)		
Faculty of Science		
Board of Studies in Mathematics		
U.G. Second Year Programme	Exit Degree	U.G. Diploma in Mathematics
Semester	III & IV	
From the Academic Year	2025-26	

University of Mumbai



(As per NEP 2020)

Sr. No.	Heading	Particulars
1	Title of program O: _____	B.A./B.Sc. (Mathematics)
2	Exit Degree	U.G. Diploma in Mathematics
3	Scheme of Examination R: _____	NEP 40% Internal 60% External, Semester End Examination Individual Passing in Internal and External Examination
4	Standards of Passing R: _____	40%
5	Credit Structure R. SU-530C(III) R. SU-530D(III)	Attached herewith
6	Semesters	Sem. III & IV
7	Program Academic Level	5.00
8	Pattern	Semester
9	Status	New
10	To be implemented from Academic Year	2025-26

Sd/-

Sign of the BOS
Chairman
Prof. B.S. Desale
BOS in Mathematics

Sd/-

Sign of the
Offg. Associate Dean
Dr. Madhav R. Rajwade
Faculty of Science &
Technology

Sd/-

Sign of the Offg. Dean
Prof. Shivram S. Garje
Faculty of Science &
Technology

Preamble

1) Introduction

The University of Mumbai has brought into force the revised syllabi as per the National Education Policy (NEP 2020) for the First year B. Sc/ B. A. Programme (Certificate Course) in Mathematics from the academic year 2024-2025. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of S.Y.B. Sc. (diploma course) Mathematics. The present syllabi of S. Y. B. Sc./ S. Y. B. A. for Semester III and Semester IV have been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics, and are exposed to rigorous methods gently and slowly. The syllabi of S. Y. B. Sc./ S. Y. B. A. would consist of two semesters and each semester would comprise of four major courses for S. Y. B. Sc. / S. Y. B. A. Mathematics. These courses contain analysis, calculus, linear algebra, IKS (related with Mathematics), differential equations, numerical methods, statistics and practical course based on them. These courses develop strong logical thinking of learner and all these are having various applications in many recent trends of science and technology and practical component provides learner with hands-on experience in applying the theoretical concepts learned in all above courses and develops computation skill of learner.

2) Aims and Objectives

- 1) Give the students a sufficient knowledge of fundamental principles, methods, and a clear perception of in numerous powers of mathematical ideas and tools and know how to use them by modelling, solving, and interpreting.
- 2) Reacting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.
- 3) Enhancing students' overall development and to equip them with mathematical modelling abilities, problem solving skills, creative talent, and power of communication necessary for various kinds of employment.
- 4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

3) Learning Outcomes

1. Real Analysis: This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole.
2. Linear Algebra: This course introduces students to the foundational concepts of vector spaces, linear transformations, and matrices. Through theoretical exploration and problem-solving, students develop a solid understanding of linear independence, span, eigenvalues, eigenvectors and related properties. This course serves as a cornerstone for further studies in various fields such as mathematics, physics, engineering and computer science.

3. Ordinary Differential Equations: This course focuses on the theory and application of ordinary differential equations (ODEs). Students learn to analyze, solve, and interpret solutions to differential equations using analytical techniques.
4. Multivariable Calculus: This course extends the study of calculus to functions of several variables. In this course, through theoretical exploration and problem-solving, students develop a comprehensive understanding of multivariable calculus and its applications in various fields.
5. Indian Mathematics: This course gives the knowledge of the significant contributions of ancient Indian mathematicians and their profound influence on modern mathematics. They will be familiar with key concepts such as the development of numerals, early algebraic methods, combinatorics, and advancements in geometry, trigonometry, and calculus as seen in the works of Aryabhata, Brahmagupta, Bhaskaracharya. Additionally, students will develop analytical skills by exploring ancient problem-solving techniques and appreciate the historical context and cultural heritage of Indian mathematical traditions.

Under Graduate Diploma in Mathematics

Credit Structure (Sem. III & IV)

(B.A./B. Sc.)- Major & Minor

	R. SU-530C(III)									
Year (Level)	Semester	Subject M1	Subject M2	Subject M3	Open Electives Related to other faculty	VSC, SEC Related to core	AEC, VEC, IKS	OJT, FP, CEP, RP Related to core	Minimum credits for the year (Sem)	Cumulative minimum credits required for award of Certificate/Diploma/Degree
2 5	III	Real Analysis and Linear Algebra - I (2) Indian Mathematics (IKS) (2) P-3 Real Analysis and Linear Algebra - I (2)	4		4	SEC: 2 JAVA Programming (2)	AEC: 2	CC: 2 CEP/FP : 2	44 (22 + 22)	UG Diploma 88
	R. SU-530D(III)									
	IV	Multivariable Calculus and Ordinary Differential Equation (2), Linear Algebra - II (2), P-4 Multivariable Calculus, Linear Algebra II and Ordinary Differential Equations (2)	4		4	VSC:2 Advanced Python (2) OR Introduction to Scilab (2)	VEC: 2	CC: 2 CEP/FP : 2		
	Cum Cr.	20	16	8	10	6+6	6+4+2	6+4		
Exit option: Award of UG Diploma in Major with 88 credits and additional 4 credits core NSQF course/ Internship OR continue with Major										

[Abbreviation - OE – Open Electives, VSC – Vocation Skill Course, SEC – Skill Enhancement Course, (VSEC), AEC – Ability Enhancement Course, VEC – Value Education Course, IKS – Indian Knowledge System, OJT – on Job Training, FP – Field Project, CEP – Community Engagement Project, CC – Co-Curricular, RP –

Sem. - III

Vertical – 1 Major

Syllabus
B.A./ B.Sc. (Mathematics)
(Sem.- III)
(MAJOR)

Name of the Course: Real Analysis and Linear Algebra I

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	Real Analysis and Linear Algebra finds extensive applications in diverse fields such as Physics, Chemistry, Biotechnology, Engineering, among others. This course aims to instill a deep understanding of Mathematical Analysis as it forms a rigorous foundation for Calculus. Learners will explore properties of Real Numbers, delve into concepts like Series and Riemann integration of functions. To provide practical context, the course incorporates applications of integration, offering students a broader perspective on the diverse uses of acquired knowledge.
2	Vertical:	Major
3	Type:	Theory
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	30 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): This course provides an introduction to advanced concepts in analysis and linear algebra with a strong emphasis on rigor. It aims to prepare students for more advanced courses in abstract analysis and linear algebra. The focus of the course is on developing formal proof skills, which not only deepens comprehension of the subject but also extends to broader applications in mathematics. CO1: Provide a solid understanding of fundamental principles and methods, equipping students with the skills to apply mathematical ideas and tools through modeling, solving, and interpretation. CO2: Illustrate the expansive nature of the subject by fostering the acquisition of essential mathematical tools for continued studies across various scientific fields. CO3: Foster students' comprehensive development by placing emphasis on problem-solving skills, nurturing creative talents, and enhancing communication abilities, all of which are vital for a range of employment opportunities. CO4: Ensure exposure to both global and local issues within the realm of Mathematical Sciences, allowing learners to explore diverse aspects of the discipline.	
8	Course Outcomes (OC): After completion of the course, students will be able to OC1 Understand and remember the concepts such as convergence/ divergence of series, Riemann Integration, beta-gamma functions, rank of matrix and related results. OC2: Apply the formulae and concepts to solve the examples related to series, Riemann Integral, area between two curves, system of linear equations etc. OC3: Analyze the convergence and divergence of series and integrability of given function and examine and interpret the conditions of consistency of systems of linear equations,	

	subspaces of vector spaces, and equivalence of row rank and column rank. OC4: Justify/ check the integrability of function, absolute and conditional convergence of series. OC5: Construct counter examples related to absolutely convergent/ divergent series, non-integrable functions consistent and inconsistent systems, elementary matrices, subspaces, and related to properties of vector spaces.
9	Modules: - Module 1: Infinite Series, Riemann Integration and Its Applications (15 Lectures) 1. Infinite series in $\mathbb{R}$. Definition of convergence and divergence. Basic examples including geometric series. Elementary results such as if $\sum_{n=1}^{\infty} a_n$ is convergent then $a_n \rightarrow 0$ but converse is not true. Cauchy Criterion (without proof), Algebra of convergent series (Statement only) and related examples. 2. Tests for convergence: Comparison Test, Limit Comparison Test (without proof), Ratio Test (without proof), Root Test (without proof), Examples, p- series test (only statement). 3. Alternating series. Leibnitz's Test. Examples. Absolute convergence, absolute convergence implies convergence but not conversely. Conditional Convergence. 4. Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower sums for a bounded real valued function on a closed and bounded interval. Riemann integrability and the Riemann integral. 5. Criterion for Riemann integrability (without proof). Characterization of the Riemann integral as the limit of a sum. (without proof). Examples. 6. Algebra of Riemann integrable functions (without proof). Also, basic results such as if (i) $f:[a,b] \rightarrow \mathbb{R}$ is integrable, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (without proof) (ii) f is integrable and $\left \int_a^b f(x)dx \right \leq \int_a^b f (x)dx$ (iii) If $f(x) \geq 0$ for all $x \in [a,b]$ then $\int_a^b f(x)dx \geq 0$ 7. Riemann integrability of a continuous function. Integrability of a bounded function whose set of discontinuities has only finitely many points (without proof). Riemann integrability of monotone functions. 8. First and Second Fundamental Theorems of Calculus (without proof). 9. Gamma and Beta functions and their properties. Relationship between them (without proof). Module 2: System of Linear Equations, Matrices and Vector Spaces (15 Lectures) 1. Systems of homogeneous and non-homogeneous linear equations, Geometric and algebraic understanding of the solutions, Elementary row and column operations. Row reduction (of a matrix to its row echelon form). Gaussian elimination. Applications of solving systems of linear equations. Examples. 2. Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as a square matrix is invertible if and only if its row echelon form is invertible and invertible matrices are products of elementary matrices. 3. Notion of row rank and the column rank with examples. Equivalence of the row rank and the column rank (without proof). 4. Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution [viz., the rank of the coefficient matrix equals the rank of the augmented matrix $[A B]$]. Equivalence of statements (without proof) (in which A denotes an $n \times n$ matrix) such as the following. (i) The system $AX = b$ of non-homogeneous linear equations has a unique solution. (ii) The system $AX = 0$ of homogeneous linear equations has no nontrivial solution.

	(iii) A is invertible. (iv) $\det A = 0$. (v) $\text{rank}(A) = n$ 5. Definition of a vector space over $\mathbb{R}$. Subspaces, criterion for a nonempty subset to be a subspace of a vector space. Examples of vector spaces, Intersections, union and sums of subspaces. Linear combination of vectors. Linear span of a subset of a vector space. Linear dependence and independence of subsets of a vector space.							
10	Recommended Reference Books: 1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer. 2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970. 3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998. 4. T. Apostol; Calculus Vol. 2; John Wiley. 5. Elementary Linear Algebra, Howard Anton and Chris Rorres, 11th Edition, Wiley, 2013. 6. Introduction to Linear Algebra, Serge Lang, 2nd Edition, Springer, 1986. 7. Linear Algebra: A Geometric Approach, S. Kumaresan, Prentice-Hall of India, 2000. 8. Linear Algebra Done Right by Sheldon Axler, 3rd Edition, Springer, 2015.							
11	Additional Reference Books 1. Ajit Kumar, S. Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014 2. D. Somasundaram and B. Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996. 3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994. 4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer. 5. R.G. Bartle and D. R. Sherbert; Introduction to Real Analysis Second Ed.; John Wiley, New York, 1992. 6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998. 7. Linear Algebra with Applications by Gareth Williams, 6th Edition, Jones and Bartlett Publishers, 2008. Sheldon Axler, Linear Algebra done right, Springer. 8. Matrix Theory by David W. Lewis, World Scientific Publishing Company, 1991.							
	<u>Scheme of the Examination</u>							
	The performance of the learners shall be evaluated in two parts. • Internal Continuous Assessment of 20 marks. • Semester End Examination of 30 marks. • A separate head of passing is required for internal and semester-end examinations.							
12	Internal Continuous Assessment: 40%	Semester End Examination: 60%						
13	Continuous Evaluation through: Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc. (at least 3)							
	<table border="1"> <thead> <tr> <th>Sr. No.</th><th>Particulars</th><th>Marks</th></tr> </thead> <tbody> <tr> <td>1</td><td>A class test of 10 marks is</td><td>10</td></tr> </tbody> </table>	Sr. No.	Particulars	Marks	1	A class test of 10 marks is	10	
Sr. No.	Particulars	Marks						
1	A class test of 10 marks is	10						

		to be conducted during each semester in an Offline mode.		
	2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05	
	3	Seminar/ group presentation on any one topic related to the syllabus.	05	
	Paper pattern of the Test (Offline Mode with One hour duration): Q1: Definitions/Fill in the blanks/ True or False with Justification. (04 Marks: 4 x 1). Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 x 3)			

14	Format of Question Paper: The semester-end examination will be of 30 marks of one hour duration covering the entire syllabus of the semester.			
	Note: Attempt any TWO questions out of THREE.			
	Q. No. 1	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
	Q. No. 2	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
	Q. No.3	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: Indian Mathematics

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	The course is designed to have glimpses of the vast mathematical knowledge that Indians had in ancient/medieval times. The learner is encouraged to learn, understand and practice the different methods given by Indian mathematicians for solving various problems. From basic Arithmetic and Geometry to the higher Math concepts like Combinatorics and Calculus, the contribution of Indian mathematicians is notable and the learner is expected to develop justified pride about their own ancestors and gain some motivation towards furthering the subject by contributing via research.
2	Vertical:	Major
3	Type:	Theory
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	30 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): This course provides an introduction to the work of Indian mathematicians and its relevance in today's world. It aims to provide knowledge to students about contribution and innovations of Indian Mathematicians. This course is designed with following objectives. CO1: To introduce students to the significant mathematical contributions of ancient Indian scholars, including Aryabhata, Brahmagupta, Bhaskara and Madhava. CO2: To study mathematical concepts found in Vedic texts, including the Sulba Sutras, and their applications in geometry, algebra, and number theory. CO3: To analyze the development of the Indian decimal number system, place value notation, and the invention of zero. CO4: To understand the mathematical principles used in Indian astronomy and their applications in architecture and engineering.	
8	Course Outcomes (OC): After completion of the course, students will be able to OC1: understand and recall the methods of obtaining square roots and cube roots, results related radius and diameter and the contributions of Indian Mathematicians OC2: explain Pythagorean triplets as appeared in Shulbasootras, impossibility of square root of negative numbers, expressed by Indian mathematicians, Varga-Sankramana, etc. OC3: apply Indian ancient methods to find squares and cubes, volume of a sphere given by Bhaskaracharya, volume of pyramid given by Brahmagupta etc. OC4: analyse the problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya, the problem of Varga Prakriti and the method given by Bhaskaracharya. OC5: create counter examples Pythagorean triplets.	
9	Modules:- Module 1: Arithmetic, Algebra and Combinatorics	

	1. The Zero and the Decimal System: The early appearance of Zero 2. Terms for the multiples of ten like 10, 20, 30 etc. in Rigveda. Terms for the higher powers of 10, given by <i>Aryabhat</i> , <i>Mahaviracharya</i> and <i>Bhaskaracharya</i> 3. The elementary operations like addition, subtraction, multiplication, division. Operations with fractions. Operations with zero. Squares and Cubes. Methods to obtain square roots and cube roots, given by <i>Aryabhat</i> and <i>Bhaskaracharya</i> . Impossibility of square root of negative numbers, expressed by Indian mathematicians. Varga-Sankramana, Quadratic Equation 4. Trairashik, Vyasta-Trairashik, Paanchrashik, Saaptarashik 5. The problem of Kuttaka and the methods given by <i>Brahmagupta</i> and <i>Bhaskaracharya</i> . The problem of Varga Prakriti and the method given by <i>Bhaskaracharya</i> . 6. Progressions and Series. 7. Combinatorics as in <i>Pingala's Chhanda:shastra</i> and <i>Bhaskaracharya's Ankpaash</i>							
	Module 2: Geometry, Trigonometry, Calculus and Astronomy							
	1. Area of triangle. Area of rectangle. Area of cyclic quadrilateral given by <i>Brahmagupta</i> . Area of rhombus, parallelogram given by <i>Bhaskaracharya</i> . Area of trapezium by <i>Bhaskaracharya</i> 2. Circumference and area of a circle. The value of pi as given by <i>Aryabhat</i> , and as appeared in Shulba-sootras. Results related to radius and diameter 3. Volume of a sphere given by <i>Bhaskaracharya</i> , Volume of pyramid given by <i>Brahmagupta</i> . Circumference of ellipse 4. Pythagoras theorem as given by <i>Aryabhat</i> . Pythagorean triplets as appeared in Shulbasootras. The "sine-value" table as given by <i>Aryabhata</i> 5. Rudiments of Calculus. <i>Madhava's</i> Infinite series for sine, cosine, arctangent and pi 6. Contribution towards Astronomy							
	Additional/Further Reading 1. History of Indian Math and mathematicians 2. Expressing numbers in Indian tradition							
10	Text Books 1. A History of Mathematics, by Carl Boyer. 2. History of science and technology in India, by Dr. Binod Bihari Satpathy 3. Mathematics in India, by Kim Plofker							
11	Reference Books 1. Aryabhateeya of Aryabhata, edited by Kripa Shankar Shukla. 2. Brahmasphutsiddhant, edited by Acharyavara Ram Swaroop Sharma. 3. Siddhantshiromani of Bhaskaracharya, by Dr. Arkasomayaji							
	<u>Scheme of the Examination</u>							
	The performance of the learners shall be evaluated in two parts. - Internal Continuous Assessment of 20 marks. - Semester End Examination of 30 marks. - A separate head of passing is required for internal and semester-end examinations.							
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Sr. No.	Particulars	Marks						
1	A class test of 10 marks is to be conducted during each	10						

		semester in an Offline mode.		
	2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05	
	3	Seminar/ group presentation on any one topic related to the syllabus.	05	
	Paper pattern of the Test (Offline Mode with One hour duration): Q1: Definitions/Fill in the blanks/ True or False with Justification. (04 Marks: 4 x 1). Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)			
14	Format of Question Paper: The semester-end examination will be of 30 marks of one hour duration covering the entire syllabus of the semester.			
	Note: Attempt any TWO questions out of THREE.			
	Q. No. 1	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
	Q. No. 2	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
	Q. No.3	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: P-3 Real Analysis and Linear Algebra- I

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	Problem-solving is a fundamental aspect of any Mathematics course. While advanced courses often emphasize the theoretical nature of the subject, engaging in problem-solving reinforces concepts and enhances learners' ability to analyze existing problems and devise solutions. This activity not only motivates learners but also empowers them to formulate new results, propose conjectures, and develop innovative theories.
2	Vertical:	Major
3	Type:	Practical
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	60 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): This course emphasizes on problem solving and motivates to think on the basic concepts of Algebra and Analysis with rigour and prepares students to study further courses. CO1. To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting. CO2. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. CO3. To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment. CO4. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	Course Outcomes (OC): After completion of the course, students will be able to OC1: Apply the formulae and concepts to solve the examples related to series, Riemann Integral, area between two curves, Gaussian elimination method etc. OC2: Analyze the convergence and divergence of series and integrability of given function and explore the fundamental properties of vector spaces and subspaces, including their intersections, unions, sums, and direct sums. OC3: Justify/ check the integrability of function, absolute and conditional convergence of series and examine and evaluate linear combinations, linear spans, and linear dependence and independence in vector spaces. OC4: Construct counter examples related to absolutely convergent/ divergent series, non-integrable functions etc. and formulate and validate results related to system of non-homogeneous linear equations and application of Cramer's rule.	
9	Modules: - Module 1: Practical for Real Analysis (30 Hours)	
	1.	Convergent and divergent series and algebra of convergent series.
	2.	Comparison and limit comparison test.
	3.	Ratio test and root test.

	4.	Alternating Series and p-series test.		
	5.	Absolute and conditional convergence.		
	6.	Upper sum and lower sum.		
	7.	Riemann integral and its properties.		
	8.	Fundamental Theorems of Calculus.		
	9.	Area between two curves, lengths of plane curves and surface area of surfaces of revolution.		
	10.	Beta and Gamma functions.		
	Module 2: Practical for Linear Algebra I (30 Hours)			
	1.	System of homogeneous and non-homogeneous linear equations		
	2.	Gaussian elimination method		
3.	Elementary row (column) operations and elementary matrices			
4.	Row space, column space, row rank and column rank			
5.	System of linear equations (using determinants) and Cramer's rule			
6.	Vector spaces and subspaces			
7.	Intersection, union, sum and direct sum of subspaces			
8.	Linear combinations and linear span of a subset			
9.	Linear independence and dependence			
10.	Basis and dimension of vector spaces			
10	Recommended Reference Books:			
	1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer.			
	2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970.			
	3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998.			
	4. T. Apostol; Calculus Vol. 2; John Wiley.			
	5. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition.			
	6. Serge Lang, Introduction to Linear Algebra, Springer.			
11	Additional Reference Books			
	1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014			
	2. D. Somasundaram and B. Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.			
	3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994.			
	4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.			
	5. R.G. Bartle and D. R. Sherbert; Introduction to Real Analysis Second Ed. ; John Wiley, New Yorm, 1992.			
	6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.			
	7. S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.			
	8. Sheldon Axler, Linear Algebra done right, Springer.			
	9. Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.			
10. David W. Lewis, Matrix theory.				

Scheme of the Examination

12	Internal Continuous Assessment: 40%	Semester End Examination: 60%												
13	Continuous Evaluation through: Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc. (at least 3) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: center;">Sr. No.</th><th style="text-align: center;">Particulars</th><th style="text-align: center;">Marks</th></tr> <tr> <td style="text-align: center;">1</td><td>Objective question test</td><td style="text-align: center;">10</td></tr> <tr> <td style="text-align: center;">2</td><td>Overall performance</td><td style="text-align: center;">05</td></tr> <tr> <td style="text-align: center;">3</td><td>Viva</td><td style="text-align: center;">05</td></tr> </table> Paper pattern of the Test (Offline Mode): Q1: (Attempt any 5 from 8) Multiple choice questions. (10 marks: 5 × 2) Duration: 1Hrs While setting question paper four MCQ on module 1 and four MCQ on module 2 both.	Sr. No.	Particulars	Marks	1	Objective question test	10	2	Overall performance	05	3	Viva	05	
Sr. No.	Particulars	Marks												
1	Objective question test	10												
2	Overall performance	05												
3	Viva	05												
14	Format of Question Paper: Scheme of examination: At the end of the Semester III, Practical examinations of three hours duration and 30 marks shall be conducted based on both the modules. Paper pattern: The question paper shall have two questions. <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">Q. No. 1</td><td>Five out of Eight multiple choice questions (four from module 1 and four from module 2) (OC1 to OC3)</td><td style="text-align: center;">Marks (3 × 5 = 15 Marks)</td></tr> <tr> <td style="text-align: center;">Q. No.2</td><td>Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)</td><td style="text-align: center;">(5 × 2 = 10 Marks)</td></tr> </table> Marks for Journals: For both Module 1 and Module 2 1. Journal: 5 marks (2.5 marks for each module 1 & module 2) The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.		Q. No. 1	Five out of Eight multiple choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)	Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)						
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Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)												

Vertical – 4 (SEC)

Name of the Course: JAVA Programming (SEC)

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	This course is designed to provide a comprehensive introduction to the Java programming language. Java is a versatile, high-level, and object-oriented programming language widely used in various applications, including web development, mobile applications, and large-scale enterprise systems. This course will cover the fundamental concepts of Java, including basic syntax, data types, control structures, object-oriented programming (OOP), and key features such as inheritance and exception handling. Additionally, students will be introduced to basic graphics programming to create simple graphical applications. Whether you are new to programming or transitioning from another language, this course will equip you with the knowledge and skills to develop basic Java programs and understand core programming principles.
2	Vertical:	Skill Enhancement Course
3	Type:	Practical
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	60 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): CO1: Understand the fundamental concepts of Java programming, including data types, variables, operators, and control structures (loops, conditionals, etc.). CO2: Apply object-oriented programming (OOP) principles such as classes, objects, encapsulation, inheritance, and polymorphism. CO3: Develop Java programs that implement inheritance to create hierarchical relationships between classes. CO4: Handle errors and exceptions effectively using Java's exception handling mechanisms. CO5: Explore basic graphical programming in Java to create simple graphical user interfaces (GUIs). CO6: Gain hands-on experience through coding exercises, developing the ability to write, compile, and run Java applications. CO7: Build a strong foundation in Java, preparing for more advanced topics and further learning in software development.	
8	Course Outcomes (OC): Upon successful completion of the course, students will be able to: OC1: Apply Java's basic syntax, control structures, and standard libraries to write and analyze programs effectively. OC2: Analyze and design Java programs that efficiently handle operations on arrays, matrices, and strings, applying appropriate algorithms for problem-solving. OC3: Perform object-oriented programming concepts such as encapsulation, inheritance, and polymorphism in Java applications.	

	• OC4: Design interactive applications using Java's built-in libraries by creating and implementing basic graphics and graphical user interfaces.								
9	Modules: - Module 1: Introduction to Java Programming (30 Lectures) <table> <tr> <td>1.</td><td> Introduction to Java programming. a) Object oriented programming (OOPs) approach: Different types of programming approach, basic concept of object-oriented programming (OOPs) approach like objects and classes, Data Abstraction, Data Encapsulation, Inheritance, polymorphism, benefits OOPs. b) Introduction to Java: History of Java, features of Java, Java environment, Writing a simple java program with output (Using <i>sytem.out.println()</i> or similar functions) and input (using Scanner class methods <i>nextInt()</i>, <i>nextFloat()</i>, <i>nextLine()</i>). c) Basic of Java: Java tokens, keywords, literals, constants, backslash character constants (program to illustrate the use of all types of backslash characters), different data types used in Java (a program to illustrate methods to input all types of data and printing them), variables (declaration and assigning values) (a program to illustrate defining different types of variables, assigning and displaying the value stored in it), type casting and its types, </td></tr> <tr> <td>2.</td><td> Basic of Java programming: a) Operators and expression: Arithmetic, Relational, logical, assignment, increment and decrement operators, conditional operators (programs to illustrate the use of each type of operators) b) Java control statements: if, if else, if else if, else statements. Switch statement (programs to illustrate all control statements). c) Java Loop statements: for loop, while loop and do..while loop (programs to illustrate use of all types of loop statements in Java). d) Use of break and continue statements in loops (programs to illustrate break and continue statements). </td></tr> <tr> <td>3.</td><td> Arrays in Java: a) Defining one dimensional array, assigning and accessing its elements, programs like finding maximum, minimum value in array, sorting of array, finding mean, median and mode of data. b) Defining two-dimensional array, assigning and accessing its element. Programs like writing a matrix using two-dimensional array, its transpose, matrix addition, matrix multiplication, determinant of 2×2 and 3×3 matrices. c) Creating string, accessing characters in string, basic string methods (programs to illustrate creation of string, displaying its each characters, string operations, and modification of string using string methods) </td></tr> </table> Module 2: Object oriented programming in Java and Java Applets (30 Hours) <table> <tr> <td>1.</td><td> Class and objects: a) Class, defining member variables and member methods, creating objects of the class, accessing members variables and member methods using objects (programs to declare more than one classes with member variables, member methods, access these members using different objects of class). Method overloading (programs to illustrate method overloading). Use of <i>this</i> operator (programs to illustrate <i>this</i> operators) b) Constructors (default and parametrized), calling another constructor, constructors overloading. Use of <i>this</i> operator in constructors. Constructors overloading (programs to illustrate each aspect of </td></tr> </table>	1.	Introduction to Java programming. a) Object oriented programming (OOPs) approach: Different types of programming approach, basic concept of object-oriented programming (OOPs) approach like objects and classes, Data Abstraction, Data Encapsulation, Inheritance, polymorphism, benefits OOPs. b) Introduction to Java: History of Java, features of Java, Java environment, Writing a simple java program with output (Using <i>sytem.out.println()</i> or similar functions) and input (using Scanner class methods <i>nextInt()</i>, <i>nextFloat()</i>, <i>nextLine()</i>). c) Basic of Java: Java tokens, keywords, literals, constants, backslash character constants (program to illustrate the use of all types of backslash characters), different data types used in Java (a program to illustrate methods to input all types of data and printing them), variables (declaration and assigning values) (a program to illustrate defining different types of variables, assigning and displaying the value stored in it), type casting and its types,	2.	Basic of Java programming: a) Operators and expression: Arithmetic, Relational, logical, assignment, increment and decrement operators, conditional operators (programs to illustrate the use of each type of operators) b) Java control statements: if, if else, if else if, else statements. Switch statement (programs to illustrate all control statements). c) Java Loop statements: for loop, while loop and do..while loop (programs to illustrate use of all types of loop statements in Java). d) Use of break and continue statements in loops (programs to illustrate break and continue statements).	3.	Arrays in Java: a) Defining one dimensional array, assigning and accessing its elements, programs like finding maximum, minimum value in array, sorting of array, finding mean, median and mode of data. b) Defining two-dimensional array, assigning and accessing its element. Programs like writing a matrix using two-dimensional array, its transpose, matrix addition, matrix multiplication, determinant of 2×2 and 3×3 matrices. c) Creating string, accessing characters in string, basic string methods (programs to illustrate creation of string, displaying its each characters, string operations, and modification of string using string methods)	1.	Class and objects: a) Class, defining member variables and member methods, creating objects of the class, accessing members variables and member methods using objects (programs to declare more than one classes with member variables, member methods, access these members using different objects of class). Method overloading (programs to illustrate method overloading). Use of <i>this</i> operator (programs to illustrate <i>this</i> operators) b) Constructors (default and parametrized), calling another constructor, constructors overloading. Use of <i>this</i> operator in constructors. Constructors overloading (programs to illustrate each aspect of
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	constructors) c) Finalize methods, abstract classes and abstract methods. Different types of class access modifier.
2.	Inheritance in Java: a) Inheritance and its types, super and sub class, <i>extends</i> keyword (programs to illustrate inheritance between two or more classes). Subclass constructor, use of <i>super</i> keyword (program to illustrate <i>super</i> keyword), method overriding (program to illustrate method overriding), final variables, final methods and final classes. Concept of interface. b) Exception handling in Java: Types of error in java program, exception, common types of exception, Need for Exception Handling, Exception Handling techniques: try and catch, multiple catch statements, finally block, usage of throw and throws. Concept of packages (programs to illustrate exception handling in java).
3.	Applets programming in Java: a) Applet and difference between applet and application program, creating applets, applet life cycle. b) Basic of HTML, designing webpage, applet tag, passing parameters to applet, getting input from user (programs to illustrate creation and running of applets in HTML tag) c) Font class. (program to display different fonts) d) Graphic class, drawing of basic shape, drawing of lines, circles, arcs, ellipse, arcs, rectangle etc. color methods. (Programs to illustrate to make different figures, filled with different colors)

List of Practical

Module 1: Basic of Java Programming.

Practical 1: Basic input output programs in Java

Practical 2: Declaring and accepting values in variables of different type, programs to illustrate the concept of type casting.

Practical 3: Programs to illustrate different types of operators used in Java.

Practical: 4: Programs to illustrate control statement if ... else if ... else.

Practical 5: Programs to illustrate control statement switch ... case.

Practical 6: Programs to illustrate for loop in Java

Practical 7: Programs to illustrate while loop and do...while loop.

Practical 8: Programs to create one dimensional array, finding maximum, minimum of array, sorting of array.

Practical 9: Programs to create two-dimensional array, some basic matrix operations using two-dimensional array.

Practical 10: Programs to illustrate string input, slicing of string.

Module 2 Object oriented programming in Java and Applet Programming.

Practical 1: Creation of class and its object, accessing class members using objects.

Practical 2: Programs to illustrate the concept of method overloading in Java and use of this operator.

Practical 3: Creation of constructor, constructor overloading, use of this operators in constructors.

Practical 4: Programs to illustrate finalize methods, abstract classes and abstract methods.

Practical 5: Programs to illustrate the concept of inheritance in Java.

Practical 6: Programs to illustrate the concept of method overriding and use of super keyword.

	Practical 7: Programs to illustrate the concept of exception handling in Java. Practical 8: Programs to create simple applets, passing values to applets. Practical 9: Programs to display text in different fonts, size and colors. Practical 10: Programs to create basic shape using java applets.													
10	Recommended Reference Books: 1. Programming with Java: a Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill. 2. Java the complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill													
11	Additional Reference Books 1. Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE5 Tutorial, Pearson Education, Third Edition 2003. 2. Ivan Bayross, Web Enabled Commercial Applications Development using Java 2, BPB Publications. Revised Edition, 2006. 3. Joe Wiggles worth and Paula Mc Millan, Java Programming: Advanced Topics, Thomson Course Technology (SPPD), Third Edition 2004.													
	<u>Scheme of the Examination</u>													
	The performance of the learners shall be evaluated in two parts. • Internal Continuous Assessment of 20 marks. • Semester End Examination of 30 marks. • A separate head of passing is required for internal and semester-end examinations.													
12	Internal Continuous Assessment: 40%	Semester End Examination: 60%												
13	Continuous Evaluation through: Quizzes, Class Tests, presentation, project, role play, creative writing, assignment etc. (at least 3) Mid semester practical examination of 20 marks will be conducted on covered syllabus (at least 50% of total syllabus) of one hour duration as per the following pattern. <table border="1" data-bbox="268 1469 845 1845"> <thead> <tr> <th>Sr. No.</th><th>Title</th><th>Marks</th></tr> </thead> <tbody> <tr> <td>1.</td><td>Quiz comprising of MCQs (Attempt any 5 out of 8) (Online/Offline)</td><td>05</td></tr> <tr> <td>2.</td><td>Class Test comprising of Problems/ Programs (Attempt any 2 out of 4)</td><td>10</td></tr> <tr> <td>3.</td><td>Viva</td><td>05</td></tr> </tbody> </table>		Sr. No.	Title	Marks	1.	Quiz comprising of MCQs (Attempt any 5 out of 8) (Online/Offline)	05	2.	Class Test comprising of Problems/ Programs (Attempt any 2 out of 4)	10	3.	Viva	05
Sr. No.	Title	Marks												
1.	Quiz comprising of MCQs (Attempt any 5 out of 8) (Online/Offline)	05												
2.	Class Test comprising of Problems/ Programs (Attempt any 2 out of 4)	10												
3.	Viva	05												
14	Format of Question Paper: The performance of the learners shall be evaluated into two parts. • Internal Continuous Assessment of 20 marks. • Semester End Examination of 30 marks. • Separate head of passing is required for internal, and semester end practical examination.													

Semester End Practical Examination (30 marks):

Semester end practical examination of 30 marks **on entire syllabus** will be conducted of three hours duration as per the following pattern.

Sr. No.	Title	Marks
1.	Problems/ Programs (Attempt any 5 out of 8)	25 Marks
2.	Journal	05 Marks

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Sem. – IV

Vertical – 1 Major

Syllabus
B.A./ B.Sc. (Mathematics)
(Sem.- IV)
(MAJOR)

**Name of the Course: Multivariable Calculus and Ordinary
Differential Equations**

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	Multivariable calculus and Differential Equation find extensive applications in diverse fields such as Physics, Chemistry, Biotechnology, Engineering, and more. This course seeks to provide learners with a comprehensive understanding of Multivariable Calculus, building upon a rigorous foundation laid by Mathematical Analysis. Through the exploration of various properties of derivatives of scalar fields and vector fields, students will gain valuable insights into the analytical aspects of Multivariable Calculus. Problem-solving skills in ordinary differential equation will enhance understanding of real world applications.
2	Vertical:	Major
3	Type:	Theory
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	30 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): This course aims to equip students with a comprehensive understanding of functions of several variables, the principles of differentiation for scalar and vector fields in multivariable calculus and also this course gives introduction to basic concepts and methods of solving differential equations and prepares students to study further courses in differential equation. CO1: To develop the understanding of vectors in $\mathbb{R}^n$ focusing on $\mathbb{R}^2$ and $\mathbb{R}^3$, acquire proficiency in working with real-valued functions of several variables and to give sufficient knowledge of basic concepts and methods of solving differential equations and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting. CO2: To demonstrate competence in analyzing neighborhoods in $\mathbb{R}^n$ and applying concepts of limits and continuity to scalar fields. CO3: To define and compute partial and directional derivatives of scalar fields, focusing on $\mathbb{R}^2$ and $\mathbb{R}^3$, and understand the Mean Value Theorem for scalar fields and to enhance students' overall development, problem solving skills, creative talent and power of communication are necessary for various kinds of employment. CO4: To explore the basic properties of differentiability, such as continuity at a point, existence of partial derivatives, and differentiability when partial derivatives exist and are continuous. CO5: To utilize concept of differentiation for practical applications, including the understanding of tangent planes and maxima-minima. CO6: To understand higher-order partial derivatives and their applications, including the Mixed Partial Derivatives Theorem, Taylor's Theorem for twice continuously differentiable functions,	

	the Method of Lagrange Multipliers and the Second Derivative Test for functions of two variables.
8	Course Outcomes (OC): After completion of the course, students will be able OC1: understand and remember the concepts such as Euclidean spaces, norm, inner product, limit, continuity, derivatives of scalar fields various methods of solving higher order linear ordinary differential equations etc. OC2: apply first and second derivative tests to find extreme values of scalar fields and the methods of solving linear differential equations with constant coefficients. OC3: verify the relationship between Differentiability and Continuity, directional derivative and continuity etc. OC4: check differentiability and continuity of scalar and vector fields and evaluate the complementary function and particular integral of given ordinary linear differential equations. OC5: create counter examples related to continuity and differentiability, directional derivative and continuity, partial derivatives, total derivative and ordinary differential equation for the given solution etc.
9	Modules: - Module 1: Continuity and Differentiability of Scalar Fields and Applications (15 Lectures) 1 Review of vectors in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and basic notions such as addition and scalar multiplication, inner product, length (norm) and distance between two points. 2 Real-valued functions of several variables (Scalar fields). Graph of a function. Level sets (level curves, level surfaces, etc). Examples. Vector valued functions of several variables (Vector fields). Component functions. Examples. 3 Sequence in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and their limits. Neighbourhoods in $\mathbb{R}^n$. Limits and continuity of scalar fields. Algebra of limits and continuity (without proofs). Iterated limits. 4 Definitions of partial derivative and directional derivative of scalar fields (with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$). Mean Value Theorem of scalar fields. 5 Differentiability of scalar fields (in terms of linear transformation). The concept of (total)derivative. Uniqueness of total derivative of a differentiable function at a point, basic results such as (i) continuity at a point of differentiability, (ii)existence of partial derivatives at a point of differentiability and (iii) differentiability when the partial derivatives exist and are continuous. 6 Gradient. Relation between total derivative and gradient of a function. Chain rule (without proof). Geometric properties of gradient. Tangent planes. 7 Euler's Theorem, Higher order partial derivatives. Mixed Partial Derivatives Theorem (n=2) (without proof). 8 Taylor's Theorem for twice continuously differentiable functions (without proof). 9 The maximum and minimum rate of change of scalar fields. Notions of local maxima, local minima and saddle points. First Derivative Test. Examples. Hessian matrix. Second Derivative Test for functions of two variables (statement only). Examples. Module 2: Homogeneous and Non-homogeneous Higher Order Linear Differential Equations (15 Lectures) (a) The general n-th order linear differential equation, linear independence of solutions of LDE, existence and uniqueness theorem (Statement only), Wronskian, classification of D.E.: homogeneous and non-homogeneous, general solution of homogeneous and non-homogeneous LDE, the differential operator and its properties.

	(b) Higher order homogeneous linear differential equations with constant coefficients, the auxiliary equations, roots of the auxiliary equations: real and distinct, real and repeated, complex and complex repeated. (c) Non-homogeneous equations: The inverse differential operator and particular integral, evaluation of $\frac{1}{f(D)}$ for the functions like e^{ax}, $\sin ax$, $\cos ax$, x^m, $x^m \sin ax$ (without proof), $x^m \cos ax$ (without proof), $e^{ax}V$ (without proof) and xV (without proof) where V is any function of x. (d) The method of variation of parameters.							
10	Recommended Reference Books: 1. T. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley. 2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer. 3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc. 4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer. 5. D. Somasundaram and B. Choudhary; A First Course in Mathematical Analysis, Narosa New Delhi, 1996. 6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994. 7. George F. Simmons, Differential Equations with Applications and Historical Notes, Taylor's and Francis, Third Edition, 2017. 8. E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.							
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	<u>Scheme of the Examination</u>							
	The performance of the learners shall be evaluated in two parts. • Internal Continuous Assessment of 20 marks. • Semester End Examination of 30 marks. • A separate head of passing is required for internal and semester-end examinations.							
12	Internal Continuous Assessment: 40%	Semester End Examination: 60%						
13	Continuous Evaluation through: Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc. (at least 3) <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 10%;">Sr.</th><th style="width: 60%;">Particulars</th><th style="width: 30%;">Marks</th></tr> </thead> <tbody> <tr> <td> </td><td> </td><td> </td></tr> </tbody> </table>		Sr.	Particulars	Marks			
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	No.		
	1	A class test of 10 marks is to be conducted during each semester in an Offline mode.	10
	2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05
	3	Seminar/ group presentation on any one topic related to the syllabus.	05
Paper pattern of the Test (Offline Mode with One hour duration): Q1: Definitions/Fill in the blanks/ True or False with Justification. (04 Marks: 4 x 1). Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)			
14	Format of Question Paper: The semester-end examination will be of 30 marks of one hour duration covering the entire syllabus of the semester.		
Note: Attempt any TWO questions out of THREE.			
Q. No. 1	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q. No. 2	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q. No.3	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: Linear Algebra - II

Sr. No.	Heading	Particulars
1	Description of the course: Including but not limited to:	This course offers a comprehensive introduction to linear transformations, eigenvalues, eigenvectors, inner product spaces, and matrix diagonalization. Topics include null spaces, images, Rank-Nullity Theorem, eigenspaces, the characteristic polynomial, and applications of the Cayley-Hamilton Theorem. Students will learn about norms, orthogonality, the Gram-Schmidt process, and matrix diagonalization, including the Spectral Theorem for real symmetric matrices and quadratic forms. These concepts have real-world applications in computer graphics, signal processing, machine learning, quantum mechanics, and optimization, equipping students to tackle practical problems in science and engineering.
2	Vertical:	Major
3	Type:	Theory
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	30 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO):	The course aims to equip students with a comprehensive understanding of linear transformations and inner product spaces, focusing on key elements like eigenvalues, eigenvectors, orthogonalization, diagonalization orthogonal diagonalization, and quadratic forms. It introduces diverse techniques for analyzing linear transformations, enhancing students' numerical aptitude in linear algebra through the application of ranks, matrices. Furthermore, the course emphasizes the development of proficiency in inner product spaces, ensuring a deep grasp of foundational concepts. CO1: Develop a solid understanding of linear transformations and their properties, providing the foundation for advanced applications in various fields. CO2: Explore the concepts of eigenvalues and eigenvectors, understanding their significance in linear transformations and matrices. CO3: Apply the Rank-Nullity Theorem to relate the rank and nullity of linear transformations, connecting algebraic and geometric perspectives. CO4: Achieve proficiency in expressing linear transformations through matrix representation and understand how alterations in bases affect matrices, facilitating a more profound comprehension of the topic. CO5: Comprehensive understanding of inner product spaces, orthogonality, and diagonalization, with applications in real-world problems such as conic sections and quadratic forms.
8	Course Outcomes (OC):	After completion of the course, students will be able to OC1: Understand basis of vector space, linear transformations, kernel, image, rank, nullity, associated matrices, inner product spaces, orthogonality of vectors and diagonalization of matrix.

	OC2: Apply the Cayley-Hamilton theorem to find inverse, power of matrix and Gram-Schmidt orthogonalization process to find orthogonal/orthonormal sets. OC3: Analyse diagonalizable matrices, orthogonally diagonalizable matrices, conic sections, and verify linear isomorphism, rank-nullity theorem for linear transformations, Cauchy-Schwarz inequality, triangle inequality. OC4: Evaluate kernel, image, eigenvalues, eigenvectors, algebraic multiplicity, geometric multiplicity, angle between vectors and orthogonal complement of subspace. OC5: Construct linear isomorphism between given vector spaces, non-diagonalizable matrix, quadratic forms and matrix with given eigenvalues.
9	Modules: - Module 1: Basis, Linear Transformations, Eigenvalues and Eigenvectors (15 Lectures) 1. Basis of a vector space and Examples, Dimension of a vector space. Bases of a vector space as a maximal linearly independent sets and as minimal generating sets (without proof). Dimension of sum of subspaces 2. Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. 3. Null-space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem and Examples. 4. Matrix associated with linear transformation of $T: V \rightarrow W$, where V and W are finite dimensional vector spaces over $\mathbb{R}$, Linear isomorphisms. Effect of change of bases on matrices of linear operator. 5. Characteristic polynomial. Properties of characteristic polynomials (only statements). Examples. Eigenvalues and eigenvectors of a linear transformation of a vector space into itself and square matrices. The eigenvectors corresponding to distinct eigenvalues of a linear transformation are linearly independent. Eigen spaces. Algebraic and geometric multiplicity of an eigenvalue. Cayley-Hamilton Theorem (Statement Only) and Applications. Module 2: Inner Product Space, Orthogonality and Diagonalization (15 Lectures) 1. Inner product spaces (over $\mathbb{R}$). Examples, Norm associated to an inner product. Cauchy-Schwarz inequality. Triangle inequality. 2. Angle between two vectors. Orthogonality of vectors. Pythagoras theorem, Orthogonal sets, Orthonormal sets. Gram-Schmidt orthogonalization process. Orthogonal basis and orthonormal basis for a finite-dimensional inner product space. 3. Orthogonal complement of any set of vectors in an inner product space. Direct Sum, Orthogonal decomposition of an inner product space with respect to its subspace. Orthogonal projection of a vector onto a line (one dimensional subspace). 4. Diagonalizable matrix. A real square matrix A is diagonalizable if and only if there is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A. $A_{n \times n}$ is diagonalizable if and only if sum of algebraic multiplicities is equal to sum of geometric multiplicities of all the eigenvalues of $A_{n \times n}$ (statement only). Procedure for diagonalizing a matrix. 5. Spectral Theorem for Real Symmetric Matrices (Statement only). Examples of orthogonal diagonalization of real symmetric matrices. Applications to quadratic forms and classification of conic sections.
10	Recommended Reference Books: 1. Elementary Linear Algebra, Howard Anton and Chris Rorres, 11th Edition, Wiley, 2013. 2. Introduction to Linear Algebra, Serge Lang, 2nd Edition, Springer, 1986. 3. Linear Algebra: A Geometric Approach, S. Kumaresan, Prentice-Hall of India, 2000.

	4. Linear Algebra Done Right by Sheldon Axler, 3rd Edition, Springer, 2015. 5. Linear Algebra with Applications by Gareth Williams, 6th Edition, Jones and Bartlett Publishers, 2008. Sheldon Axler, Linear Algebra done right, Springer. 6. Matrix Theory by David W. Lewis, World Scientific Publishing Company, 1991.														
	<u>Scheme of the Examination</u>														
	The performance of the learners shall be evaluated in two parts. • Internal Continuous Assessment of 20 marks. • Semester End Examination of 30 marks. • A separate head of passing is required for internal and semester-end examinations.														
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Q. No.3	Module 1 and 2	Attempt any THREE out of FOUR . (Each question of 5 marks)	(a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks	

Name of the Course: P-4 Multivariable Calculus, Linear Algebra II and Ordinary Differential Equations

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	Problem solving forms one of the basic aspects of any course in Mathematics. Higher courses in Mathematics focus mainly on the theoretical nature of the subject, nevertheless, the problem- solving activity strengthens the concepts and helps the learners develop their ability to think over the existing problems in the subject, and also to create and crack new problems! This way a learner is not just motivated, but elevated also, to formulate new results, suggest new postulates (usually known as conjectures), and design new theories.
2	Vertical:	Major
3	Type:	Practical
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	60 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): This course introduces basic concepts of Calculus, Linear Algebra and differential equation with rigour and prepares students to study further courses. CO1. To give sufficient knowledge of fundamental principles, methods, and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting. CO2. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. CO3. To enhance students' overall development, problem solving skills, creative talent, and power of communication, which are necessary for various kinds of employment. CO4. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	Course Outcomes (OC): After completion of the course, students will be able OC1: apply first and second derivative tests to find extreme values of scalar fields, compute eigenvalues and eigenvectors, apply the Cayley-Hamilton theorem, and understand inner product spaces and related inequalities and also apply the various methods to solve ordinary linear differential equations. OC2: verify the relationship between Differentiability and Continuity, directional derivative and continuity etc. OC3: check differentiability and continuity of scalar and vector fields and evaluate the complementary function and particular integral of given ordinary linear differential equations. OC4: create counter examples related to continuity and differentiability, directional derivative and continuity, partial derivatives and total derivative etc and construct orthogonal and orthonormal sets using the Gram-Schmidt process and compute orthogonal complements of subspaces.	

9

Modules: -**Module 1: Practical for Multivariable Calculus and Ordinary Differential Equations (30 Hours)**

1.	Limits and continuity of scalar fields, using "definition and otherwise", iterated limits.
2.	Directional derivatives, partial derivatives and mean value theorem of scalar fields.
3.	Differentiability of scalar field and Total derivative.
4.	Gradient, level sets and tangent planes.
5.	Chain rule, higher order partial derivatives and mixed partial derivatives of scalar fields.
6.	Maximum and minimum rate of change of scalar fields. Finding Hessian/Jacobian matrix.
7.	Taylor's Theorem.
8.	Finding maxima, minima and saddle points. Second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
9.	Wronskian and linear independence of solutions.
10.	Higher order homogeneous linear differential equations with constant coefficients.

Module 2: Practical for Ordinary Differential Equations and Linear Algebra II (30 Hours)

1.	Evaluation of particular integral for $X = e^{ax}$.
2.	Evaluation of particular integral for $X = \sin ax, \cos ax$.
3.	Evaluation of particular integral for $X = x^m, x^m \sin ax, x^m \cos ax$.
4.	Evaluation of particular integral for $X = e^{ax}V$ and $X = xV$ where V is any function of x .
5.	Method of undetermined coefficients.
6.	Method of variation of parameters.
7.	Linear Transformations, Matrix Representation of Linear Transformations and Isomorphisms.
8.	Eigenvalues, Eigenvectors, Cayley-Hamilton Theorem and its Applications
9.	Gram-Schmidt Orthogonalization Process
10.	Diagonalization of Matrices and Quadratic Forms

10

Text Books

1. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley.
2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer.
3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc.
4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer.
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8. E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.

	9. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition. 10. Serge Lang, Introduction to Linear Algebra, Springer. 11. S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning. 12. Sheldon Axler, Linear Algebra done right, Springer. 13. Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers. 14. David W. Lewis, Matrix theory.													
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14**Format of Question Paper:****Scheme of examination:**

At the end of the Semester IV, Practical examinations of three hours duration and 30 marks shall be conducted based on both the modules.

Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2

1. Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Vertical – 4 (VSC)

Name of the Course: **Advanced Python (VSC)**

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	This course will equip undergraduate students with essential skills in numerical and scientific computing using Python, preparing them for careers in data science, engineering, and applied sciences. It focuses on four essential libraries— NumPy (for array-based numerical computations), SciPy (for advanced scientific functions including problem solving in mathematics), Pandas (for data manipulation and analysis) and Matplotlib (for creating visual representations of data). Students will gain practical experience in using four libraries to solve real-world problems, including numerical equation solving, performing matrices and system of equations, and applying statistical methods for data analysis. The course emphasizes the application of these techniques to data science, showing how numerical computing can be applied to fields such as engineering, economics, and research. Additionally, the course covers basic statistical analysis to help interpret data and solve real-world data science problems. By the end of the course, students will have the skills needed to efficiently perform numerical computations, analyze complex datasets, and visualize data effectively, making this course a valuable foundation for anyone working in data science or related fields.
2	Vertical:	Vocational Skill Course
3	Type:	Practical
4	Credits:	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	Hours Allotted:	60 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): By the end of the course, students will: CO1: Understand the fundamental concepts of numerical and scientific computing and its role in solving mathematical problems using Python. CO2: Apply SciPy for solving scientific problems involving linear algebra, optimization, and statistics. CO3: Learn to use Pandas for data analysis and manipulation of large datasets. CO4: Create visualizations using Matplotlib to represent data and scientific computation graphically. CO5: Gain hands-on experience by working on practical exercises that apply the theory to real-world problems.	
8	Course Outcomes (OC): After completion of the course, students will be able. OC1: Apply NumPy, SciPy, Pandas, Matplotlib functions to solve numerical, statistical,	

	optimization problems and System of equations. OC2: Analyze clear and insightful data visualizations using these packages. OC3: Perform numerical computations on multi-dimensional arrays using these packages. OC4: Design programs for effective data manipulation, visualization, and analysis of small datasets.				
9	Modules: - Module 1: Numerical Computing with Python (30 Lectures) <table border="1"> <tr> <td></td><td> 1. Introduction to Data Analysis: Data Analysis: Understanding the Nature of the Data, the data analysis process including Problem definition, Data extraction, Data cleaning, Data transformation, Data exploration, Predictive modelling, Model, validation/test, Visualization and interpretation of results, Deployment of the solution, Quantitative and Qualitative Data Analysis 2. Review of Python: Python Interpreter, IPython Notebook, Anaconda distributor, Google Colab, Introduction to Jupyter Notebooks and installation, Modules in python. 3. Vectors, Matrices, and Multidimensional Arrays with NumPy: Importing modules through the NumPy Library, NumPy Array objects, creating arrays, Indexing, slicing, and reshaping arrays, Vectorized expressions including arithmetic operations, operations on arrays, matrix and vector operations. Problems on Array manipulations, mathematical operations with NumPy, Reading and Writing Array Data on Files 4. Data Processing and Analysis with Pandas: Introduction to pandas, Data Structures a) Series – Declaring series, Selecting the Internal Elements, Assigning Values to the Elements, Defining Series from NumPy Arrays and Other Series, Filtering Values, Evaluating Values, NaN Values, Series as Dictionaries, Operations between Series b) DataFrame - Defining a DataFrame, Selecting Elements, Assigning Values, Membership of a Value, deleting a Column, Filtering, DataFrame from Nested dict, Transposition of a DataFrame, indexing </td></tr> </table> Module 2: Scientific Computing with Python (30 Lectures) <table border="1"> <tr> <td></td><td> 1. Reading and Writing Data with Pandas- I/O API Tools- readers and writers, CSV and Textual Files, Introduction to The Seaborn Graphics Library 2. Plotting and Visualization with Matplotlib: Introduction to data visualization, Matplotlib architecture, Pyplot, Use of the kwargs, Creating line plots, scatter plots, bar charts, and histograms, Customizing plots: titles, labels, legends, and styles 3. Scientific Computation with SciPy: Introduction to the SciPy library, Optimization, Overview of SymPy, a) Matrix operations: addition, multiplication, transpose, inverse, Solving System of </td></tr> </table>		1. Introduction to Data Analysis: Data Analysis: Understanding the Nature of the Data, the data analysis process including Problem definition, Data extraction, Data cleaning, Data transformation, Data exploration, Predictive modelling, Model, validation/test, Visualization and interpretation of results, Deployment of the solution, Quantitative and Qualitative Data Analysis 2. Review of Python: Python Interpreter, IPython Notebook, Anaconda distributor, Google Colab, Introduction to Jupyter Notebooks and installation, Modules in python. 3. Vectors, Matrices, and Multidimensional Arrays with NumPy: Importing modules through the NumPy Library, NumPy Array objects, creating arrays, Indexing, slicing, and reshaping arrays, Vectorized expressions including arithmetic operations, operations on arrays, matrix and vector operations. Problems on Array manipulations, mathematical operations with NumPy, Reading and Writing Array Data on Files 4. Data Processing and Analysis with Pandas: Introduction to pandas, Data Structures a) Series – Declaring series, Selecting the Internal Elements, Assigning Values to the Elements, Defining Series from NumPy Arrays and Other Series, Filtering Values, Evaluating Values, NaN Values, Series as Dictionaries, Operations between Series b) DataFrame - Defining a DataFrame, Selecting Elements, Assigning Values, Membership of a Value, deleting a Column, Filtering, DataFrame from Nested dict, Transposition of a DataFrame, indexing		1. Reading and Writing Data with Pandas- I/O API Tools- readers and writers, CSV and Textual Files, Introduction to The Seaborn Graphics Library 2. Plotting and Visualization with Matplotlib: Introduction to data visualization, Matplotlib architecture, Pyplot, Use of the kwargs, Creating line plots, scatter plots, bar charts, and histograms, Customizing plots: titles, labels, legends, and styles 3. Scientific Computation with SciPy: Introduction to the SciPy library, Optimization, Overview of SymPy, a) Matrix operations: addition, multiplication, transpose, inverse, Solving System of
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linear equations, Square and rectangular Systems

b) Statistics- Review of Statistics and Probability, importing stats module in SciPy, compute the mean, median, variance and standard deviation of a dataset, random numbers.

List of Practical

	Module1: Numerical Computing with Python
1	Practical based on NumPy array objects, creating arrays , Indexing, slicing, and reshaping arrays.
2	Practical based on vectorized expressions including arithmetic operations, operations on arrays.
3	Practical based on matrix and vector operations.
4	Practical based on array manipulations, mathematical operations with NumPy.
5	Practical based on reading and Writing Array Data on Files.
6	Practical based on declaring series, selecting the Internal Elements, Assigning Values to the Elements.
7	Practical based on defining Series from NumPy Arrays and other Series, Filtering Values, Evaluating Values, NaN Values.
8	Practical based on series as Dictionaries, Operations between Series.
9	Practical based on defining a DataFrame, Selecting Elements, Assigning Values, Membership of a Value, deleting a Column, Filtering.
10	Practical based on DataFrame from Nested dict, Transposition of a DataFrame, indexing.
	Module2: Scientific Computing with Python
1	Practical based on reading and writing Data with Pandas, readers and writers, CSV and Textual Files.
2	Practical based on data visualization with Matplotlib, Pyplot.
3	Practical based on creating line plots, scatter plots with Matplotlib.
4	Practical based on creating bar charts, and histograms with Matplotlib.
5	Practical based on Customizing plots: titles, labels, legends, and styles with Matplotlib.
6	Practical based on Matrix operations: addition, multiplication with SciPy.
7	Practical based on Matrix operations: transpose, inverse with SciPy.
8	Practical based on Solving System of linear equations, Square and rectangular Systems with SciPy.
9	Practical based on importing stats module in SciPy, to compute the mean and median of dataset.
10	Practical based on to compute variance and standard deviation of a dataset, random numbers.

10 Recommended Reference Books:

1. Robert Johansson Numerical Python: Scientific Computing and Data Science Applications with Numpy, SciPy and Matplotlib Second Edition Apress Publ.
2. Fabio Nelli Python Data Analytics: Data Analysis and Science Using Pandas, matplotlib, and the Python Programming Language Apress Publ
3. Vijay Kotu and Bala Deshpande Data Science Concepts and Practice Second Edition Morgan Kauffman Publication
4. Bernd Klein Data Analysis with Python Numpy, Matplotlib and Pandas

11 Additional Reference Books

1. Joel Grus Data Science from Scratch O'Reilly publication

2. Wes McKinney - Python for Data Analysis Data Wrangling with pandas, NumPy, and Jupyter-O'Reilly Media (2022)

3. Alberto Boschetti Luca Massaron Python Data Science Essentials Third Edition Packt Publishing 2018

4. Eli Bressert SciPy and NumPy O'Reilly Media Publication

5. Gaël Varoquaux, Emmanuelle Gouillart, Olaf Vahtras, Pierre de Buyl Scipy Lecture Notes(www.scipy-lectures.org), 2020 edition

Scheme of the Examination

- The performance of the learners shall be evaluated in two parts.
- Internal Continuous Assessment of 20 marks.
 - Semester End Examination of 30 marks.
 - A separate head of passing is required for internal and semester-end examinations.

12 Internal Continuous Assessment: 40%

Semester End Examination: 60%

13 Continuous Evaluation through: Quizzes, Class Tests, presentation, project, role play, creative writing, assignment etc.
(at least 3)

Mid semester practical examination of 20 marks will be conducted on **covered syllabus (at least 50% of total syllabus)** of one hour duration as per the following pattern.

Sr. No.	Title	Marks
1.	Quiz comprising of MCQs (Attempt any 5 out of 8) (Online/Offline)	05
2.	Class Test comprising of Problems/ Programs (Attempt any 2 out of 4)	10
3.	Viva	05

14 Format of Question Paper:

- The performance of the learners shall be evaluated into two parts.
- Internal Continuous Assessment of 20 marks.
 - Semester End Examination of 30 marks.
 - Separate head of passing is required for internal, and semester end practical examination.

Semester End Practical Examination (30 marks):

Semester end practical examination of 30 marks **on entire syllabus** will be conducted of three hours duration as per the following pattern.

Sr. No.	Title	Marks
1.	Problems/ Programs (Attempt any 5 out of 8)	25 Marks
2.	Journal	05 Marks

The students are required to perform 75% of the Practical for the journal to be duly certified.
The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Name of the Course: Introduction to Scilab

Sr. No.	Heading	Particulars
1	Description the course: Including but not limited to:	Scilab is open and free software for scientific calculation. It provides numerical, programming and graphics environment. It can be run using a variety of operating system UNIX, Windows, Linux etc. makes learners to understand concepts more clearly and learners can solve problems of greater complexity with ease.
2	Vertical:	VSC
3	Type:	Practical
4	Credits:	2 credits
5	Hours Allotted:	60 Hours
6	Marks Allotted:	50 Marks
7	Course Objectives (CO): This course provides an introduction to mathematical programming using open software Scilab. CO1: Provide a solid understanding of input output using mathematical Scilab. CO2: Illustrate think like a programmer and developer in various scientific fields. CO3: Foster student's comprehensive development in logical thinking. CO4: Ensure exposure to Mathematical Sciences, allowing learners to explore diverse aspects of the discipline.	
8	Course Outcomes (OC): After completion of the course, students will be able to. OC1: Perform basic mathematical operations using Scilab software. OC2: Analyze different types of data using plotting functions in Scilab software. OC3: Execute loops and conditional statements using Scilab software. OC4: Find solutions of problems based on Numerical Analysis.	
9	Modules: - Module 1. Introduction to Scilab, the general environment, The editor, Command Window, graphics window, window management and workspace customization, Variables assignments, display array in terms of matrices and vectors, Displaying output data, data file, Scilab functions. Module 2. Relational and logical operators, Branching Statements and program design, Loops, the while loop, for loop, Tests, 2D and 3D plotting, developing the skills of writing a program Solving differential equations.	

List of Practical

	Module 1	
1	Basic mathematical operations in Scilab.	
2	Find roots of a given polynomial, form a polynomial when roots are given.	
3	Check whether the given number is positive, negative or zero.	
4	Matrix and vector operations.	
5	Solution of system of linear equations.	
6	Find quotient and remainder when a positive integer divides an integer.	
7	Sum of first n natural numbers.	
8	Sum of digits of a positive integer.	
9	Reverse the digits of a positive integer.	
10	Programme to find gcd of two integers.	
	Module 2.	
1	Factorial of a number.	
2	Convert a positive integer to binary form.	
3	Plot 2D graphs.	
4	Plot 3D graphs.	
5	Generate Fibonacci sequence upto given number of terms.	
6	Root of equation using Bisection method.	
7	Root of equation using Newton Raphson method.	
8	Numerical Integration using Trapezoidal rule.	
9	Numerical Integration using Simpson's rule.	
10	Solution of differential equation using Runge Kutta method.	

10 Recommended Reference Books:

1. Stephen L. Campbell, Jean-Philippe Chancelier and Ramine Nikoukhah: Modeling and Simulation in Scilab/Scicos. Springer USA, 2006.
2. Sandeep Nagar, Introduction to Scilab: For Engineers and Scientists. Apress publisher, New York, USA, 2017.
3. A.S.Nair, SCILAB (A free software to MATLAB), S. Chand Publishing, New Delhi, India, 2012.
4. Scilab for beginners. - www.scilab-enterprises.com

Scheme of the Examination

The performance of the learners shall be evaluated in two parts.

- Internal Continuous Assessment of 20 marks.
- Semester End Examination of 30 marks.
- A separate head of passing is required for internal and semester-end examinations.

11 Internal Continuous Assessment: 40%

Semester End Examination: 60%

12 Continuous Evaluation through:
Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc.
(at least 3)

Sr. No.	Particulars	Marks
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1	Objective question test	10
2	Overall performance	05
3	Viva	05

Paper pattern of the Test (Offline Mode):

Q1: (Attempt any 5 from 8) Multiple choice questions. (10 marks: 5×2)

Duration: 1Hrs

While setting question paper four MCQ on module 1 and four MCQ on module 2 both.

13 Format of Question Paper:

Scheme of examination:

At the end of the Semester III, Practical examinations of three hours duration and 30 marks shall be conducted based on both the modules.

Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks ($3 \times 5 = 15$ Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	($5 \times 2 = 10$ Marks)

Marks for Journals:

For both Module 1 and Module 2

2. Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Letter Grades and Grade Points:

Semester GPA/ Programme CGPA Semester/ Programme	% of Marks	Alpha-Sign/ Letter Grade Result	Grading Point
9.00 - 10.00	90.0 - 100	O (Outstanding)	10
8.00 - < 9.00	80.0 - < 90.0	A+ (Excellent)	9
7.00 - < 8.00	70.0 - < 80.0	A (Very Good)	8
6.00 - < 7.00	60.0 - < 70.0	B+ (Good)	7
5.50 - < 6.00	55.0 - < 60.0	B (Above Average)	6
5.00 - < 5.50	50.0 - < 55.0	C (Average)	5
4.00 - < 5.00	40.0 - < 50.0	P (Pass)	4
Below 4.00	Below 40.0	F (Fail)	0
Ab (Absent)	-	Ab (Absent)	0

Sd/-

**Sign of the BOS
Chairman
Prof. B.S. Desale.
BOS in Mathematics**

Sd/-

**Sign of the
Offg. Associate Dean
Dr. Madhav R. Rajwade
Faculty of Science &
Technology**

Sd/-

**Sign of the Offg. Dean
Prof. Shivram S. Garje
Faculty of Science &
Technology**