University of Mumbai

No. AAMS_UGS/ICC/2024-25/21

CIRCULAR:-

All the Principals of the Affiliated Colleges, Directors of the Recognized Institutions and the Head, University Departments in Faculty of Science & Technology is invited to this office Circular No. AAMS_UGS/ICC/2023-24/23 dated 08th September, 2023 relating to the NEP UG & PG Syllabus.

They are hereby informed that the recommendations made by the Board of Studies in Mathematics at its online meeting held on 20th March, 2024 and subsequently passed by the Board of Deans at its meeting held on 18th April, 2024 **vide** item No. 6.1 (N) have been accepted by the Academic Council at its meeting held on 20th April, 2024 **vide** item No. 6.1 (N) and that in accordance therewith to introduce **M.Sc. (Mathematics) (Sem – III &IV)** syllabus as per NEP 2020 as per appendix with effect from the academic year 2024-25.

(The said circular is available on the University's website www.mu.ac.in).

MUMBAI – 400 032
24th July, 2024

(Prof. (Dr.) Baliram Gaikwad)
I/c. REGISTRAR

To,

All the Principals of the Affiliated Colleges, Directors of the Recognized Institutions and the Head University Departments in Faculty of Science & Technology.

A.C/6.1/20/04/2024

Copy forwarded with Compliments for information to:-

1) The Chairman, Board of Deans,
2) The Dean, Faculty of Science & Technology,
3) The Chairman, Board of Studies in Mathematics,
4) The Director, Board of Examinations and Evaluation,
5) The Director, Department of Students Development,
6) The Director, Department of Information & Communication Technology,
7) The Director, Institute of Distance and Open Learning (IDOL Admin), Vidyanagari,

Copy forwarded for information and necessary action to :-	
1	The Deputy Registrar, (Admissions, Enrolment, Eligibility and Migration Dept)(AEM), dr@eligi.mu.ac.in
2	The Deputy Registrar, Result unit, Vidyanagari drresults@exam.mu.ac.in
3	The Deputy Registrar, Marks and Certificate Unit,. Vidyanagari dr.verification@mu.ac.in
4	The Deputy Registrar, Appointment Unit, Vidyanagari dr.appointment@exam.mu.ac.in
5	The Deputy Registrar, CAP Unit, Vidyanagari cap.exam@mu.ac.in
6	The Deputy Registrar, College Affiliations & Development Department (CAD), deputyregistrar.uni@gmail.com
7	The Deputy Registrar, PRO, Fort, (Publication Section), Pro@mu.ac.in
8	The Deputy Registrar, Executive Authorities Section (EA) eau120@fort.mu.ac.in He is requested to treat this as action taken report on the concerned resolution adopted by the Academic Council referred to the above circular.
9	The Deputy Registrar, Research Administration & Promotion Cell (RAPC), rapc@mu.ac.in
10	The Deputy Registrar, Academic Appointments & Quality Assurance (AAQA) dy.registrar.tau.fort.mu.ac.in ar.tau@fort.mu.ac.in
11	The Deputy Registrar, College Teachers Approval Unit (CTA), concolsection@gmail.com
12	The Deputy Registrars, Finance & Accounts Section, fort draccounts@fort.mu.ac.in
13	The Deputy Registrar, Election Section, Fort drelection@election.mu.ac.in
14	The Assistant Registrar, Administrative Sub-Campus Thane, thanesubcampus@mu.ac.in
15	The Assistant Registrar, School of Engg. & Applied Sciences, Kalyan, ar.seask@mu.ac.in
16	The Assistant Registrar, Ratnagiri Sub-centre, Ratnagiri, ratnagirisubcentar@gmail.com

Copy for information :-	
1	P.A to Hon'ble Vice-Chancellor, vice-chancellor@mu.ac.in
2	P.A to Pro-Vice-Chancellor pvc@fort.mu.ac.in
3	P.A to Registrar, registrar@fort.mu.ac.in
4	P.A to all Deans of all Faculties
5	P.A to Finance & Account Officers, (F & A.O), camu@accounts.mu.ac.in

AC-20/04/2024

Item No.6.1 (N)

As Per NEP 2020

University of Mumbai

Title of the Program

A- P. G. Diploma in Mathematics __ } **2023-24**

B- M. Sc. (Mathematics) (Two Years) } **2023-24**

C- M. Sc. (Mathematics) (One Year) - 2027-28

Syllabus for

Semester: III & IV

(Academic Year 2024-25)

Ref: GR dated 16th May, 2023 for Credit Structure of PG

University of Mumbai

(As per NEP 2020)

Sr. No	Heading		Particulars	
01	Title of the program			
	O:________________A		A	P. G. Diploma in Mathematics
	O:________________B		B	M. Sc. (Mathematics) (Two Years)
	O:________________C		C	M. Sc. (Mathematics) (One Year)
02	Eligibility	O:__________A	A	B.Sc. Mathematics (with major)/B. Sc. General (with mathematics course up to third year)/B. E./B. Tech. Graduation in Mathematics with a level 5.5
		O:__________B	B	
		O:__________C	C	Graduation with 4 year U. G. Degree (Honours/Honours with Research) with specialization in Mathematics or equivalent academic level 6.0 OR Graduate with four year U. G. Degree program with maximum credits required for award of minor degree is allowed to take up to the post graduate programme in minor subject provided the student has acquired the required number of credits as prescribed by the concerned Board of studies.
03	Duration of the program R:_________________		A	1 Year
			B	2 Years
			C	1 Year
04	Intake capacity R:_________________		150	

05	Scheme of Examination R:____________________	NEP 50% Internal 50% External, Semester End Examination Individual Passing in Internal and External Examination	
06	Standards of Passing R:____________________	40%	
07	Credit Structure R:____________________	Attached herewith	
08	Semesters	A	Sem. I & II
		B	Sem. I, II, III & IV
		C	Sem. I & II
09	Program Academic Level	A	6.0
		B	6.5
		C	6.5
10	Pattern	Semester	
11	Status	New	
12	To be implemented from Academic Year	A	2023-24
		B	
		C	2027-28

Sign of Chairman BOS

Prof. B. S. Desale

Head, Department of Mathematics

Sign of Dean

Name of the Dean: Prof. S. S. Garje

Dean, Science and Technology

Preamble

1. Introduction

With reference to GR No. NEP – 2022/ Pr. Kr. 09/ Vishi -3 Shikana Higher and Technical Education, Government of Maharashtra dated 16th May 2023, University of Mumbai has adopted New Education Policy 2020 for the Post Graduate Departments. Accordingly the revised academic curricula and syllabi are being brought into force from the academic year 2023–24. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of applications of Mathematics to real world problems has been increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the University of Mumbai has prepared the syllabus of M. Sc. Mathematics. The present syllabi of M. Sc. is for Semester I and Semester II have been designed as per U.G.C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly.

2. Aims & Objectives

1. Deliver students a sufficient knowledge of fundamental principles, methods and clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.
2. Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of sciences.
3. Enhancing student's overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.
4. Student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

3. Learning Outcomes

1. Enabling students to develop positive attitude towards mathematics as an interesting and valuable subject.
2. Enhancing student's overall development and to equip them with mathematical

modeling, abilities, problem solving skills, creative talent and power of communication.

3. Acquire good knowledge and understanding in advanced areas of mathematics, science and technology.

4. Any other Point (If any)

5. Baskets of Electives

Semester I

1. 513016150611: Commutative Algebra
2. 513016150612: Graph Theory
3. 513016150613: Programming in Python
4. 513016150614: Linear and Non Linear Programming

Semester II

1. 513016251611: Algebraic Number Theory
2. 513016251612: Advanced Counting Techniques in Discrete Mathematics
3. 513016251613: Algebraic Topology
4. 513016251614: Numerical Analysis

Semester III

1. 513016355611: Advanced Partial Differential Equations
2. 513016355612: Coding Theory
3. 513016355613: Matrix Algebra
4. 513016355614: Integral Transforms
5. 513016355615: Financial Mathematics
6. 513016355616: Basics of Numerical Analysis
7. 513016355617: Quadratic Forms
8. 513016355618: Basic Algebraic Geometry
9. 513016355619: Numerical Linear Algebra
10. 513016355620: R Programming

Semester IV

1. 513016456511: Lie Algebra
2. 513016456512: Representation Theory of Finite Groups
3. 513016456513: Special Functions
4. 513016456514: Calculus on Manifolds
5. 513016456515: Calculus of Variations
6. 513016456516: Boundary Value Problems
7. 513016456517: Numerical Methods for PDE
8. 513016456518: Stochastic Calculus for Finance
9. 513016456519: Machine Learning
10. 513016456520: Computational Algebra
11. 513016456521: Design Theory

6. Credit Structure of the Program

M.Sc. Mathematics

Sr. No.	Course Code	Course	Mandatory/Elective	Theory/Practical	Credits
Semester-I (Level 6.0)					
1	5130161501	Algebra I	Mandatory	2TH + 2 PR	4
2	5130161502	Real Analysis	Mandatory	2TH + 2 PR	4
3	5130161503	Ordinary Differential Equations	Mandatory	2TH + 2 PR	4
4	5130161504	Discrete Mathematics and Number Theory	Mandatory	2 PR	2
5	5130161505	Research Methodology	Mandatory	2TH + 2 PR	4
Students have to choose any one from the following elective courses					
6	513016150611	Commutative Algebra	Elective	2TH + 2 PR	4
7	513016150612	Graph Theory	Elective	2TH + 2 PR	4
8	513016150613	Programming in Python	Elective	2TH + 2 PR	4
9	513016150614	Linear & Non Linear Programming	Elective	2TH + 2 PR	4
				Total Credits :	22

Sr. No.	Course Code	Course	Mandatory /Elective	Theory/Practical	Credits
Semester-II (Level 6.0)					
1	5130162511	Algebra II	Mandatory	2TH + 2 PR	4
2	5130162512	Topology	Mandatory	2TH + 2 PR	4
3	5130162513	Complex Analysis	Mandatory	2TH + 2 PR	4
4	5130162514	Partial Differential Equations	Mandatory	2 PR	2
5	5130162515	OJT (On Job Training)/ FP(Field Project)	Mandatory	2TH + 2 PR (At least four week training at industry or Field work Certificate)	4
Students have to choose any one from the following elective courses					
6	513016251611	Algebraic Number Theory	Elective	2TH + 2 PR	4
7	513016251612	Advanced Counting Techniques in Discrete Mathematics	Elective	2TH + 2 PR	4
8	513016251613	Algebraic Topology	Elective	2TH + 2 PR	4
9	513016251614	Numerical Analysis	Elective	2TH + 2 PR	4
				Total Credits :	22
Cumulative Credits (SEM-I and SEM-II):					44

Note: Exit will be permitted as per National Education Policy-2020 (NEP 2020) on successful completion of first year (with 44 credits) and Post Graduate Diploma in Mathematics will be awarded by the University.

Sr. No.	Course Code	Course	Mandatory /Elective	Theory/Practical	Credits
Semester-III (Level 6.5)					
1	5130163551	Algebra III	Mandatory	2TH + 2 PR	4
2	5130163552	Differential Geometry	Mandatory	2TH + 2 PR	4
3	5130163553	Measure Theory and	Mandatory	2TH + 2 PR	4
4	5130163554	Probability Theory and Statistics	Mandatory	2 PR	2
5	5130163555	Research Project	Mandatory		4
Students have to choose any one from the following elective courses					
6	513016355611	Advanced Partial Differential Equations	Elective	2TH + 2 PR	4
7	513016355612	Coding Theory	Elective	2TH + 2 PR	4
8	513016355613	Matrix Algebra	Elective	2TH + 2 PR	4
9	513016355614	Integral Transforms	Elective	2TH + 2 PR	4
	513016355615	Financial Mathematics	Elective	2TH + 2 PR	4
11	513016355616	Basics of Numerical Analysis	Elective	2TH + 2 PR	4
12	513016355617	Quadratic Forms	Elective	2TH + 2 PR	4
13	513016355618	Basic Algebraic Geometry	Elective	2TH + 2 PR	4
14	513016355619	Numerical Linear Algebra	Elective	2TH + 2 PR	4
15	513016355620	R Programming	Elective	2TH + 2 PR	4
				Total Credits :	22
Cumulative Credits (SEM-I, SEM-II and SEM-III):					66

Sr. No.	Course Code	Course	Mandatory/ Elective	Theory/Practical	Credits
Semester-IV (Level 6.5)					
1	5130164561	Algebra IV	Mandatory	2TH + 2 PR	4
2	5130164562	Fourier Analysis	Mandatory	2TH + 2 PR	4
3	5130164563	Functional Analysis	Mandatory	2TH + 2 PR	4
4	5130164564	Research Project	Mandatory		6
Students have to choose any one from the following elective courses					
5	513016456511	Lie Algebra	Elective	2TH + 2 PR	4
6	513016456512	Representation Theory of Finite Groups	Elective	2TH + 2 PR	4
7	513016456513	Special Functions	Elective	2TH + 2 PR	4
8	513016456514	Calculus on Manifolds	Elective	2TH + 2 PR	4
9	513016456515	Calculus of Variations	Elective	2TH + 2 PR	4
10	513016456516	Boundary Value Problems	Elective	2TH + 2 PR	4
11	5130164565171	Numerical Methods for PDE	Elective	2TH + 2 PR	4
12	513016456518	Stochastic Calculus for	Elective	2TH + 2 PR	4
13	513016456519	Machine Learning	Elective	2TH + 2 PR	4
14	513016456520	Computational Algebra	Elective	2TH + 2 PR	4
15	513016456521	Design Theory	Elective	2TH + 2 PR	4
				Total Credits :	22
Cumulative Credits (SEM-I, SEM-II, SEM-III and SEM-IV):					88
Note: As per NEP 2020 with successful completion of 2 years P.G. degree in M.Sc. Mathematics (with cumulative credits 88) will be awarded by the University.					

Letter Grades and Grade Points:

Semester GPA/ Program CGPA Semester/Program	% of Marks	Alpha-Sign / Letter Grade Result
9.00-10.00	90.0-100	O (Outstanding)
8.00 =< 9.00	80.0 =< 90.0	A+ (Excellent)
7.00 =< 8.00	70.0 =< 80.0	A (Very Good)
6.00 =< 7.00	60.0 =< 70.0	B+ (Good)
5.50 =< 6.00	55.0 =< 60.0	B (Above Average)
5.00 =< 5.50	50.0 =< 55.0	C (Average)
4.00 =< 5.00	40.0 =< 50.0	P (Pass)
Below 4.00	Below 40	F (Fail)
Ab (Absent)	-	Absent

Teaching Pattern and Workload

1. Two theory lectures and two Practicals per week for each four credit courses.
2. Two Practicals per week for each Two credit courses.
3. Two theory and two Practicals per week for each four credit research project.
4. Four theory and two Practicals per week for each six credit research project.
5. Practicals to be conducted as per list provided for each course. In addition there shall be tutorials, seminars as required for the content of the course.
6. Research project batch will consist of minimum 3 and maximum 8 students.

Scheme of Examination

A. For Four Credit Course

1. 50 : 50 scheme for all PG courses i. e. 50 marks for continuous internal assessment and 50 marks for end semester examination.
2. Separate head of passing is required for internal and end semester examination.
3. **Mid semester examination** shall be of 50 (40+10) marks.

Items	Marks	Duration	Remark
Internal Test (Theory component)	30	40 marks examination of 2 hours duration	Based on Unit-I and II
Internal Test (Practical component)	10		
Journal Certification and Attendance	10		At the end of semester based on all practicals as per the list provided in syllabus
Total	50		

4. The end semester examination shall be of 50 marks and 2.5 hours duration (It consist of two sections. Section-I (Theory Component 30 marks), Section-II (Practical Component 20 marks))

Question No	Unit	Theory/Practical	Max. Marks	Total maximum marks including options for each question
Section-I (Theory Component)				
Q1	Unit I and II	Theory	10	15
Q2	Unit III and IV	Theory	10	15
Q3	Common	Theory	10	15
Section-II (Practical Component)				
Q4	Unit III	Practical	10	15
Q5	Unit IV	Practical	10	15
Total			50	

B. For Two Credit Course:

25 : 25 scheme for all PG courses i.e. 25 marks for continuous assessment and 25 marks for end semester examination. Separate head of passing is required for internal and end semester examination.

Mid semester examination shall be of 25 (15+10) marks.

Items	Marks	Duration	Remark
Internal Test (Practical component)	15	15 marks examination of 1 hour duration	Based on Unit-I
Journal Certification and Attendance	10		At the end of semester based on all practicals as per the list provided in syllabus
Total	25		

The end semester examination shall be of 25 marks and 1.5 hours duration

Question No	Unit	Practical	Max. Marks	Total marks including options
Q1	Unit I	Practical	12	18
Q2	Unit II	Practical	13	20
Total Marks			25	

Note : The examination for the Elective Courses will be scheduled and conducted only for the Elective Courses offered in that particular Semester and for backlog Elective course ATKT depending on the record of students appearing for the examination in that particular semester

C. For Research Project:

1. The workload assigned for each research project of 4 and 6 credits are 4 and 6 hours per week respectively. Moreover, under the supervision of a research project guide, each student has to spend a minimum of 2 hours per week in the soft computing lab for the purpose of using e-learning resources, mathematical software's, graphics and plotting, literature review via e- database etc as a practical.
2. The evaluation of the Project submitted by a student in the form of dissertation shall be made by a Committee appointed by the Head, University Department of Mathematics / respective affiliated college.
3. The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the respective college. This committee shall have two members, possibly with one external referee and research project supervisor.
4. Exceptional project output may be displayed on the website of the University.
5. A research project can consist of different research topices depending on the students and research project supervisor.
6. The students will be evaluated on their individual performance and their contribution to the research project.
7. The Marking scheme for the project of 4 credits are detailed below:
    - I. Literature Review : 10 Marks
    - II. Contents of the project : 30 Marks
    - III. Main Output of the Project : 10 Marks
    - IV. Use of e-learning resources : 10 Marks
    - V. Presentation of the project : 20 Marks
    - VI. Viva of the project : 20 Marks
    - VII. Total Marks : 100 Marks per project per student
8. The Marking scheme for the project of 6 credits are detailed below:
    - VIII. Literature Review : 10 Marks
    - IX. Contents of the project : 60 Marks
    - X. Main Output of the Project : 20 Marks
    - XI. Use of e-learning resources : 20 Marks
    - XII. Presentation of the project : 20 Marks
    - XIII. Viva of the project : 20 Marks
    - XIV. Total Marks : 150 Marks per project per student

Scheme of evaluation R8435 for M. Sc. Mathematics:

A) 100% internal evaluation scheme for University Department of Mathematics. Both Mid and End semester examinations will be conducted by the Department and answer books will be shown to the students.

B) For affiliated PG centers end semester examination will be conducted by the University.

Syllabus Committee

(Ref: AAMS/ICD/2023-24/550 dated 02 Feb. 2024)

Sr. No.	Name	Department/ Institute	Signature
01	Prof. B. S. Desale (Chairman BOS)	University Department of Mathematics	
02	Prof. Vinayak Kulkarni (Co-ordinator)	University Department of Mathematics	
03	Dr. Anuradha Garge (Member BOS)	University Department of Mathematics	
04	Dr. Shridhar Pawar (Member BOS)	Sant Rawool Maharaj Mahavidyalay, Kudal	
05	Prof. R. P. Deore	University Department of Mathematics	
06	Prof. J. V. Prajapat	University Department of Mathematics	
07	Dr. Madhumita Gangopadhyay	University Department of Mathematics	
08	Dr. Deepak Sarwe	University Department of Mathematics	
09	Mr. Kamalakar Survade	University Department of Mathematics	
10	Mr. Shantilal Shendage	University Department of Mathematics	

M. Sc. (Mathematics) Semester III & IV
Semester-III
5130163551: Algebra III

Course Objectives:

1. To discuss about the concepts of solvability and nilpotency.
2. To develop connections between two subjects like topology and ring theory.
3. To formulate basic properties of modules.
4. To study the structure theorem for modules over principal ideal domains.

Course Outcome:

1. Students will understand about important types of groups like solvable groups and nilpotent groups and applications of these groups.
2. Students will be able to recall Zariski topology and verify properties of open and closed subsets defined therein.
3. Students will be able to state the definitions of finitely generated modules, free modules, rank of a free module.
4. Students will be able to prove the structure theorem for finitely generated modules over a principal ideal domain. They will compute the invariant factors and elementary divisors in the special case of abelian groups.

Prerequisites: Basics of ring theory and group theory.

Note: All results have to be done with proof unless otherwise stated.

Unit-I: Groups (15 hours)
A_5 is simple, Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Zassenhaus Lemma, Jordan-Hölder theorem, Direct and Semi-direct products, Examples such as (i) The group of affine transformations $x \mapsto ax + b$ as semi-direct product of the group of linear transformations acting on the group of translations.
(ii) Dihedral group D_{2n} as semi-direct product of $\mathbb{Z}_2$ and $\mathbb{Z}_n$.
Classification of groups of order 12. (Ref: M. Artin, Algebra).

Unit-II: Prime spectrum and Zariski topology (15 hours)
Nilradical definition and relation to prime ideals, Jacobson radical and maximal ideals, Radical of an ideal, Annihilator ideal, Local rings and equivalent conditions for a local ring, Prime spectrum of a ring and Zariski topology, idempotents and connectedness, ring homomorphisms and induced map on Spec. Hilbert Nullstellensatz (only statement) and its corollaries. (Ref: Atiyah and MacDonald, Introduction to Commutative Algebra, chapter 1, exercise 15, 16, 17, 18, 21 and 22).

Unit-III: Modules (15 hours)
Modules over rings, Submodules. Module homomorphisms, kernels. Quotient modules. Isomor phism theorems. (ref:D.S. Dummit and R.M. Foote, Abstract Algebra) Generation of modules, finitely generated modules, (internal) direct sums and equivalent conditions. (ref:D.S. Dummit and R.M. Foote, Abstract Algebra) Free modules, free module of rank n. For a commutative ring R, R^n is isomorphic to R^m if and only if $n = m$. Matrix representations of homomorphisms between free modules of finites ranks. (Ref: N. Jacobson, Basic Algebra, Volume 1.)
Dimension of a free module over a P.I.D. (ref: S. Lang, Algebra).

Unit-IV: Modules over principal ideal domains (15 hours)
Noetherian modules and equivalent conditions. Rank of an R-module. Torsion submodule $Tor(M)$ of a module M, torsion free modules, annihilator ideal of a submodule. (ref:D.S. Dummit and R.M. Foote, Abstract Algebra).

Finitely generated modules over a PID: If N is a submodule of free module M (over a P.I.D.) of finite rank n, then N is free of rank $m \leq n$. Any submodule of a finitely generated module over a P.I.D. is finitely generated. (ref: S. Lang, Algebra).

Structure theorem for finitely generated modules over a PID: Fundamental theorem, Exis- tence (Invariant Factor Form and Elementary Divisor Form), Fundamental theorem, Uniqueness. Applications to the Structure theorem for finitely generated Abelian groups and linear operators. (ref:D.S. Dummit and R.M. Foote, Abstract Algebra).

List of Practicals

1. Solvable groups and nilpotent groups.
2. Examples of direct and semi-direct products.
3. Properties of local rings.
4. Zariski topology and its properties.
5. Modules, submodules, quotient modules.
6. Generation of modules and their properties.
7. Noetherian modules, torsion submodule: examples and computations.
8. Interplay between invariant factor form and elementary divisor form.

Recommended Text Books:

1. D. S. Dummit and R.M. Foote, Abstract Algebra.
2. S. Lang, Algebra, Springer Verlag, 2004.
3. N. Jacobson, Basic Algebra, Volume 1, Dover, 1985.

4. M. Artin, Algebra, Prentice Hall of India.

5. M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra, Indian Edition 2007.

5130163552: Differential Geometry

Course Objectives:

1. To study geometrical concepts of translations, rotations and reflections.
2. To study the various geometrical aspects of plane and space curves in view of tangent, normal, binormal, curvature, torsion
3. To understand the concept of regular surface, smooth surface, orientable surface
4. To learn the first fundamental form and second fundamental form and their geometrical application.

Course Outcome: On completion of the course, the learner will be able to

1. Parametrize curves and surfaces.
2. Compute arc length, tangent vector, normal, binormal, curvature, torsion for plane and space curves.
3. Understand the role of the first fundamental theorem and the second fundamental theorem in the computation of Gaussian curvature, mean curvature and principal curvature.
4. Differentiate between plane curves and space curves, orientable and non orientable surfaces.

Prerequisites: Vector algebra and Analytical geometry.

Unit-I: Isometries of $\mathbb{R}^n$ (15 hours)

Review of vector geometry: lines and planes, Orthogonal transformations of $\mathbb{R}^n$ and Orthogonal matrices. Reflection, Rotations and Translations of $\mathbb{R}^2$ and $\mathbb{R}^3$, Euler's theorem, Hyperplanes, Refection map about a hyperplane W of $\mathbb{R}^n$ through the origin, Isometry of $\mathbb{R}^n$, Isometries of the plane, Orientation preserving and reversing isometries of $\mathbb{R}^n$, Glide reflection.
(References for Unit I: S. Kumaresan, Linear Algebra, A Geometric Approach and M. Artin, Algebra, PHI.)

Unit-II: Curves in Plane and Space (15 hours)

Parametrized curves, Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Curvature and torsion of curves in $\mathbb{R}^3$, Plane curves, Signed curvature for plane curves, Fundamental theorem for plane curves, Space curves, Serret-Frenet equations. Fundamental theorem for space curves.

Unit-III: Regular Surface (15 hours)
Regular surfaces in $\mathbb{R}^3$, Examples. Surfaces as graphs, Surfaces as level sets, Surfaces of revolution. Tangent space to a surface at a point, Equivalent definitions. Smooth functions on a surface, Differential of a smooth function defined on a surface. Orientable surfaces. Mobius band is not orientable.

Unit-IV: Curvature (15 hours)
The first fundamental form. Isometries of surfaces, Surface area, The Gauss map, The shape operator of a surface at a point, Self-adjointness of the shape operator, The second fundamental form, Normal curvature, Principle curvatures and directions, Euler's formula, Meusnier's Theorem, Gaussian curvature and mean curvature, Computation of Gaussian curvature. Geodesics.

List of Practicals

1. Examples based on translations, rotations and reflections.
2. Examples based on isometry and its properties.
3. Examples based on regular curves, unit speed curves, arc length of curves.
4. To find the tangent, normal, binormal, curvature and torsion of the plane and space curves and hence to verify Serret-Frenet equations.
5. Verify the given surface is a regular surface.
6. Examples based on the tangent, normal and orientability.
7. To find the first fundamental form and second fundamental form of a regular surface
8. To find the Gaussian curvature, Principal curvature, mean curvature
9. Examples based on geodesics.

Recommended Text Books:

1. M. DoCarmo, Differential geometry of curves and surfaces, Dover.
2. A. Pressley, Elementary Differential Geometry, Springer UTM.
3. C. Bar, Elementary Differential geometry, Cambridge University Press, 2010.
4. M. Artin, Algebra, PHI.
5. S. Kumaresan, Linear Algebra, A Geometric Approach.

5130163553: Measure Theory and Integration

Course Objectives:

1. Understand the concept of measure, measurable sets on abstract space and Lebesgue measure, Borel sets and Borel measure in the Euclidean space $\mathbb{R}^d$.

2. Understand the concept of measurable and Lebesgue integrable functions.

3. Analyze and apply the Monotone convergence theorem, Fatou's lemma and Lebesgue Dominated convergence theorem.

4. Understand signed measure, importance of Hahn Decomposition theorem and Radon Nikodym theorem.

Course Outcome: After successful completion of course the learner will be able to

1. Identify measurable sets and measurable functions.

2. Integrate measurable function on measurable set.

3. Apply the convergence theorems for Lebesgue integrable functions.

4. Apply the Radon Nikodym theorem.

Unit-I: Measures and Measurable Sets (15 hours)
Additive set functions, σ-algebra, Borel set, Borel algebra of $\mathbb{R}^d$. Outer measure, μ^* measurable sets (Definitions due to Carathéodory), μ^* measurable subsets of X forms a σ algebra, constructing measure from outer measure, measure space $(X, \sum, \mu)$. Lebesgue outer measure in $\mathbb{R}^d$, properties of exterior measure, monotonicity property and countable sub-additivity property of Lebesgue measure, translation invariance of exterior measure, example of set of measure zero. Measurable sets and Lebesgue measure, properties of measurable sets. Any closed subset and any open subset of $\mathbb{R}^d$ is Lebesgue measurable. Every Borel set in $\mathbb{R}^d$ is Lebesgue measurable.

[Reference for Unit I: 1. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.
2. Elias M. Stein and Rami Shakarchi, *Real Analysis, Measure Theory, Integration and Hilbert Spaces*, New Age International Limited, India]

Unit-II: Measurable functions and their Integration (15 hours)
Existence of a subset of $\mathbb{R}$ which is not Lebesgue measurable. Measurable functions on $(X, \sum, \mu)$, simple functions, properties of measurable functions. If $f \geq 0$ is a measurable function, then there exists a monotone increasing sequence (s_n) of non-negative simple measurable functions converging to point wise to the function f. Lusin's theorem. Egorov's theorem, Integral of nonnegative simple measurable functions defined on

the measure space $(X, \sum, \mu)$ and their properties. Integral of a non-negative measurable function.
[Reference for unit II: 1. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.
2. Elias M. Stein and Rami Shakarchi, *Real Analysis, Measure Theory, Integration and Hilbert Spaces*, New Age International Limited, India]

Unit-III: Convergence Theorems (15 hours)
Monotone convergence theorem. If $f \geq 0$ and $g \geq 0$ are measurable functions, then $\int(f+g)d\mu = \int f d\mu + \int g d\mu$, Fatou's lemma, summable functions, vector space of summable functions, Lebesgue's dominated convergence theorem. Lebesgue integral of bounded functions over a set of finite measure, Bounded convergence theorem.
[Reference for unit III: 1. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.
2. Royden H. L. Real Analysis, PHI]

Unit-IV: Space of Integrable functions (15 hours)
Lebesgue and Riemann integrals: A bounded real valued function on [a, b] is Riemann integrable if and only if it is continuous at almost every point of [a, b]; in this case, its Riemann integral and Lebesgue integral coincide.
Approximation of Lebesgue integrable functions by continuous functions. The space $L^1(\mu)$ of integrable functions, properties of L^1 integrable functions, Riesz-Fischer theorem.
Signed Measures, positive set, negative set and null set. Hahn decomposition theorem. Radon Nikodym theorem.

[Reference for unit IV: 1. Elias M. Stein and Rami Shakarchi, *Real Analysis, Measure Theory, Integration and Hilbert Spaces*, New Age International Limited, India
2. Royden H. L., *Real Analysis*, PHI
3. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.]

List of Practicals

1. Computation of measure of sets in $\mathbb{R}$ and $\mathbb{R}^2$
2. Problems based on measurable sets.
3. Problem based on measurable functions.
4. To calculate measure of addition, and multiplication of measurable functions.
5. Problems based on convergence of measurable functions.
6. Integration of positive, negative measurable functions.

7. Problems based on Monotone convergence theorem, Fatou's lemma.

8. Problems based on signed measures.

Recommended Text Books:

1. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.

2. Elias M. Stein and Rami Shakarchi, *Real Analysis, Measure Theory, Integration and Hilbert Spaces*, New Age International Limited, India

3. Royden H. L., *Real Analysis*, PHI.

4. Terence Tao, *Analysis II*, Hindustan Book Agency (Second Edition).

5130163554: Probability and Statistics

Course Objectives:

1. To define conditional probability, demonstrate Bayes' theorem and recognize random variables.
2. To discuss discrete random variables, distribution functions and density functions.
3. To calculate expectation, variance and characteristic functions of a random variable.
4. To derive the Chebyshev inequality, law of large numbers and to formulate the Central limit theorem.

Course Outcome:

1. Students will be able to calculate probability, conditional probability and verify independence of events.
2. They will develop the notion of random variables and distribution functions of random variables, both discrete and continuous.
3. The students will recognize the notions of expectation, variance of a random variable and its characteristic function.
4. Students will estimate using the Chebyshev inequality. They will also employ the law of large numbers and the central limit theorem to compute in specific examples.

Pre-requisites: A first course in real analysis.

Unit-I: Basic concepts of Probability and random variables (15 hours)
Classical probability spaces, basic concepts of probability, events, Conditional Probability, total probability formula, Bayes theorem, independence of events. Random variables, discrete and absolutely continuous random variables, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distrbutions, independence of random variables.
Reference: Chapters 2,7 and 8 of Capinski and Zastawniak.

Unit-II: Expectation and variance of a random variable, Characteristic functions and Limit theorems (15 hours)
Simple random variable, expectation and variance of discrete and absolutely continuous random variables, Characteristic functions of random variables, both discrete and continuous, Chebyshev inequality, Weak law of large numbers, Kolmogorov Strong law of large numbers (statement only), Central limit theorem (statement only).
Reference: Chapters 9, 11 and 12 of Capinski and Zastawniak.

List of Practicals

1. Problems based on Total probability formula, Bayes' theorem, independence of events.
2. Problems based on discrete random variables having Bernouli, Binomial, Poisson distributions and absolutely continuous random variables having normal, exponential distributions etc.
3. Expectation and variance of random variables both discrete and absolutely continuous.
4. Properties of expectation and variance.
5. Problems based on Chebyshev inequality, weak law of large numbers, strong law of large numbers and Central limit theorem.

Recommended Text Books:

1. Marek Capinski and Tomasz Zastawniak, Probability through problems.
2. J.F. Rosenthal, A first look at rigorous probability theory, World Scientific.
3. Kai Lai Chung, Farid AitSahlia, Elementary Probability theory, Springer, Verlag.
4. Sheldon M.Ross, A first course in Probability (8th edition), Pearson.

5130163555: Research Project

In order to maintain its quality and purpose of research projects, the following guidelines has to be follow

1. Research project batch will consist of minimum 3 and maximum 8 students.
2. A research project can consist of different research topices depending on the students and research project supervisor.
3. The students will be evaluated on their individual performance and their contribution to the research project.
4. Students has to do detailed literature review on their respective research topic.
5. It is mandatory to spend at least two hours a week on e-learning resources as practical work during research project.
6. Use of soft computing tools/ mathematical softwares viz Matlab, Mathematica, MathCAD etc. will be highly appreciated.
7. For semester IV, a student may be allowed to continue with the research topic done by her/him in Semester III. However the material submitted for evaluation in Semester 3 cannot be included for evaluation in Semester IV.
8. A student has option of choosing a research topic and supervisor different from the one in Semester III.

513016355611: Adavanced Partial Differential Equations

Course Objectives:

1. To study the nature of given partial differential equations viz parabolic, hyperbolic and elliptic.
2. To study the fundamental properties of the Laplace equation.
3. To study the fundamental properties of the Heat equation.
4. To study the fundamental properties of the Wave equation.

Course Outcome:

1. Students will be able to grasp nature of the differential operator viz parabolic, hyperbolic and elliptic.
2. Students will be able to understand the solution and various properties of the Laplacian operator,heatoperator and waveoperator.
3. Students will be aware about applications of the Laplacian operator,heatoperator and waveoperator.

Unit-I: Local existence theory (15 hours)
Basic preliminaries and notations, The differential operator, Real first order equations, the general Cauchy problem, Cauchy-Kowalevsky theorem, Local solvability: the Lewy example, the fundamental solution.

Unit-II: Laplace operator (15 hours)
Symmetry properties of the Laplacian, basic properties of the Harmonic functions, Green's identities, The mean value theorem, Liouville's theorem, the Fundamental solution, the Dirichlet and Neumann boundary value problems, the Green's function. Applications to the Dirichlet problem in a ball in $\mathbb{R}^n$ and in a half space of $\mathbb{R}^n$.

Unit-III: Heat operator (15 hours)
The properties of the Gaussian kernel, solution of initial value problem for heat equation: homogeneous and non-homogeneous, The fundamental solution for heat operator, Heat equation in a bounded domains, Maximum principle for the heat equation and applications.

Unit-IV: Wave operator (15 hours)
Wave operator in dimensions 1, 2 & 3; Cauchy problem for the wave equation. D'Alemberts solution,of the one dimensional wave equation, Poisson formula of spherical means, Hadamards method of descent, Inhomogeneous Wave equation, Wave equation in a bounded domain.

List of Practicals

1. Find the solution of given Cauchy Problem under the prescribed conditions.
2. Find the characteristics curve of the given partial differential equations.
3. Find the solution of the Dirichlet's problem in a half space
4. Find the solution of the Dirichlet's problem in a ball.
5. Find the solution of the heat equation using the Fourier transform on $\mathbb{R}^n \times (0, \infty)$ subjected to the non-zero initial temperature.
6. Find the solution of inhomogeneous heat equation.
7. Find the solution of one dimensional wave equation using the D'Alemberts method.
8. Find the solution of wave equation using the Hadmards descent method.
9. Find the solution of inhomogeneous wave equation.

Recommended Text Books:

1. G.B. Folland, Introduction to partial differential equations, Overseas Press.
2. F. John, Partial Differential Equations, Springer publications.
3. Lawrence C. Evans, Partial Differential Equation, Second edition, American Mathematical Society, 2010.

513016355612: Coding Theory

Course Objectives:

1. To implement coding theory algorithms in practice.
2. To categorize codes into linear and non-linear.
3. To demonstrate the importance of finite fields.
4. To plan for projects in this area in the next semester.

Course Outcome:

1. The students will be able to detect errors, do correction and decoding. They will learn about communication channels, maximum likelihood decoding, Hamming distance, nearest neighbor / minimum distance decoding and distance of a code.
2. They will be able to identify vector spaces over finite fields, linear codes and their generator matrix and parity check matrix. They will learn about cosets and syndrome decoding.
3. Students will get accustomed to the definition of cyclic codes and their properties. They will also learn about MacWilliams identities.
4. Students will develop an understanding of coding theory over finite fields using the theoretical concepts of Frobenius automorphism, trace code, subfield code and relation to projective geometry.

Prerequisites: A basic course in linear algebra and ring theory.

Unit-I: Error detection, Correction and Decoding, Linear codes (15 hours)
Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbour/minimum distance decoding, Distance of a code. Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbour decoding for linear codes, Syndrome decoding.

Unit-II: Cyclic codes (15 hours)
Definitions, Generator polynomials, Generator and parity check matrices, Hamming code, Simplex code, Decoding of cyclic codes, BCH codes, definition and parameters.

Unit-III: Codes and their duals (15 hours)
Weight distribution and MacWilliams theorem, distance distribution of a code, distance polynomial of a code, MacWilliams identities, Kravchouk polynomials, Minimum distance separable codes.

Unit-IV: Finite fields and their application to coding theory (15 hours)
Basics of finite fields, Frobenius automorphism, Trace of finite extensions of finite fields, trace code and subfield code, Delsarte's theorem, projective geometry and the main theorem, projective planes, hyperplanes, points and lines.

List of Practicals

1. Various examples of decoding.
2. Linear codes and computations of generator and parity check matrices.
3. Cyclic codes and relations to ideals.
4. Decoding of cyclic codes.
5. Examples illustrating MacWilliams theorem.
6. MDS codes: examples and Kravchouk polynomial properties.
7. Basic properties of finite fields.
8. Projective geometry and associated codes.

Recommended Text Books:

1. San Ling and Chaoing xing, Coding Theory- A First Course.
2. Rudolf Lidl and Günter Pilz, Applied Abstract Algebra, Second Edition, Springer Verlag.
3. Jurgen Bierbrauer, Introduction to Coding Theory, Chapman and Hall/CRC.
4. Joseph A. Gallian, Contemporary Abstract Algebra, Fourth Edition, Narosa.
5. Lidl and Niedeereiter, Finite fields, Cambridge University Press, 1996.

513016355613: Matrix Algebra

Course Objectives:

1. To define the basics of bilinear and quadratic forms.
2. To give examples of skew symmetric forms and derive Sylvester's theorem over real numbers.
3. To outline the structure of the general linear group over fields.
4. To explain the structure of the symplectic group over fields.

Course Outcome:

1. The students will be able to differentiate between degenerate and non-degenerate bilinear forms.
2. The students will compute the rank and signature of real matrices.
3. The students will interpret the concepts of group theory in the special context of the general linear groups over fields.
4. The students will describe the symplectic linear group over fields in terms of the elementary symplectic transvections.

Prerequisites : A basic course in Linear algebra at master's level is a must.

Unit-I: Basics of bilinear forms over fields (15 hours)
Linear functions and bilinear forms, bilinear form, left radical and right radical of a bilinear form, non-degenerate bilinear form, equivalent conditions for non-degeneracy, orthogonality relation, characterization of orthogonality in terms of the bilinear form, discriminant of a bilinear form.

Unit-II: Alternate forms over fields (15 hours)
Defintion of alternate form, Skew symmetric form, Structure theorem for alternate bilinear forms, rank of an alternate matrix over a field is even, determinant of the alternate matrix over a field is a square, Statement of Sylvester's diagonalization theorem, proof of Sylvester's theorem.

Unit-III: General linear group over fields(15 hours)
Structure of the general linear group over fields, special linear group over fields, commutator subgroup, properties of commutator subgroup, special linear group being its own commutator subgroup (except finitely many cases), elementary matrices, generation of the special linear group by elementary matrices, counting cardinalities of the general linear group and the special linear group over finite fields, computing the center of the general linear group, computing the normalizer of the subgroup of diagonal matrices in the general linear group. (Ref: Jacobson, Basic Algebra, Chapter 6).

Unit-IV: Symplectic group over fields (15 hours)
Symplectic form, symplectic basis, symplectic transvection, properties of symplectic transvections, generation of symplectic group by symplectic transvections, center of the symplectic group, commutator subgroup of the symplectic group, exceptions for coincidence of the symplectic group and its commutator subgroup.

List of Practicals

1. Computations of left and right radicals.
2. Non-degeneracy and orthogonality.
3. Rank of alternate matrix and examples of skew symmetric, alternate forms.
4. Diagonalization and Sylvester's theorem.
5. Commutator subgroups: examples and properties.
6. Computation of center and normalizer in various examples.
7. Properties of symplectic transvections.
8. Center and commutator subgroup in symplectic group.

Recommended Text Books:

1. N. Jacobson, Basic Algebra, Second edition, Dover, 1985.
2. Roger Carter, Lie Algebras of Finite and Affine Type, Cambridge University Press, January 2010.
3. Lam, T. Y, Introduction to quadratic forms over fields, Graduate Studies in Mathematics, Volume 67.
4. S. Lang, Algebra, Springer Verlag, 2004.
5. Larry C. Grove, Classical Groups and Geometric Algebra, Graduate Studies in Mathematics volume 39, American Mathematical Society, 2001 .

513016355614: Integral Transforms

Course Objectives:

1. To study the development of various kernels of the integral transforms.
2. To learn the properties and applications of the Laplace transforms.
3. To learn the properties and applications of the Fourier transforms.
4. To learn the properties and applications of the Mellin transforms.
5. To learn the properties and applications of the Hankel transforms.

Course Outcome:

1. Students will be able to grasp the concept of integral transforms and development of corresponding kernels.
2. Students will be able to understand various properties of the Laplace transform, Fourier transform, Mellin transform and Hankel tranform.
3. Students will aware about applications of the integral transform in the solution of initial and boundary value problems.

Unit I: The Laplace Transform (15 hours)
Definition of Laplace Transform, Laplace transforms of some elementary functions, Existence theorem, Properties of Laplace transform, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Convolution Theorem, Inversion theorem, Laplace transform of special functions: Heaviside unit step function, Dirac-delta function and Periodic function, Application of Laplace transform to evaluation of integrals and solutions of ODEs & PDEs: One dimensional heat equation & wave equation.
(Reference for Unit I: Section 4.1, 4.2, 4.3, 4.5, 4.6, 4.7, 5.2, 5.3, 5.4 of L. Andrews and B. Shivamogg, Integral Transforms for Engineers)

Unit II: The Fourier Transform (15 hours)
Fourier integral representation, Fourier integral theorem, Fourier Sine & Cosine integral representation, Riemann-Lebesque lemma, Fourier transform pairs, Fourier Sine & Cosine transform pairs, Fourier transform of elementary functions, Properties of Fourier Transform, Convolution Theorem, Convolution integrals of Fourier, Parseval's Identity, Cosine & Sine convolution integrals, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems, Heat conduction in solids (one dimensional problems in infinite & semi infinite domain).
(Reference for Unit II: Section 2.2, 2.3, 2.4, 2.5, 2.7, 3.2, 3.3 of L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India)

Unit III: The Mellin Transform (15 hours)
Derivation for Mellin transform & its inversion by Fourier integral theorem, Properties and evaluation of Mellin transforms, Complex variable method, Convolution theorem for Mellin transform, Applications of Mellin transform: Summation of series, Products of random variables, Boundary value problems.
(Reference for Unit III: Section 6.1, 6.2, 6.3, 6.4 of L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India)

Unit IV: The Hankel Transform (15 hours)
Definition of Hankel tarnsform and its inverse, Evaluation of Hankel transform, Hankel tarnsform of some elementary functions, Properties of Hankel tarnsform, Parseval's relation, Evaluation of integrals, Applications of Hankel tarnsform to solutions of Partial differential equations.
(Reference for Unit IV: Sections 7.1, 7.2, 7.3, 7.4 of L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India)

List of Practicals

1. Find the Laplace transform and the inverse Laplace transform of the given function.
2. Find solution of the given ordinary differential equation using the Laplace transforms.
3. Find solution of the given boundary value problem using the Laplace transforms.
4. Find the Fourier transform and the inverse Fourier transform of the given function.
5. Find solution of the given initial and boundary value problem using the Fourier transforms.
6. Find the Mellin transform of the given function.
7. Find solution of the given initial and boundary value problem using the Mellin transforms.
8. Find the Hankel transform of the given function.
9. Find solution of the given initial and boundary value problem using the Hankel transforms.

Recommended Text Books:

1. L. Andrews and B. Shivamogg, *Integral Transforms for Engineers*, Prentice Hall of India.

2. Lokenath Debnath and Dambaru Bhatta, *Integral Transforms and their Applications*, CRC Press Taylor & Francis.

3. Brian Davies, *Integral transforms and their Applications*, Springer.

4. I.N.Sneddon, *Use of Integral Transforms*, Tata-McGraw Hill.

5. R. Bracemell, *Fourier Transform and its Applications*, MacDraw hill.

513016355615: Financial Mathematics

Course Objectives:

1. To learn the various financial terms and their role in the portfolio management.
2. To understand the corelation between risk and return.
3. To study the various aspects of risk management.
4. To study the role of statistical measures in portfolio and risk management.

Course Outcome:

1. Students will be able to grasp the various concept of finance.
2. Students will be able to understand portfolio and risk management.
3. Students will aware about applications of statistical measures in portfolio and risk management.

Unit I: A Simple Market Model (15 hours)
Basic notations and assumptions, no arbitrage principle, one-step binomial model, risk and return, forward contracts, call and put options, foreign exchange, managing risk with options.

Unit II: Risk Free Assets (15 hours)
Time value of money: simple interest, periodic compounding, streams of payments, continuous compounding, comparision of compounding methods.
Money market: zero-coupon bonds, coupon bonds, money market account.

Unit III: Portfolio Management (15 hours)
Risk and return: expected return, standard deviation as risk measure.
Two securities: risk and expected return on a portfolio, feasible set.
Several securities: risk and expected return on a portfolio, minimum variance portfolio, efficient frontier, two-fund theorem, market portfolio.
Capital asset pricing model.

Unit IV: Forward and Futures Contracts (15 hours)
Forward contracts: underlying asset, forward price, value of forward contract.
Futures: Pricing, margins, hedging with futures, index futures.

List of Practicals

1. Examples based on no-arbitrage principle.
2. Examples based on risk and returns.
3. Examples based on periodic and continuous compunding.
4. Examples based on zero coupon bonds and coupon bonds.
5. Examples based on risk management.
6. Examples based on Capital asset pricing model.
7. Examples based on hedging.
8. Examples based on index futures.

Recommended Text Books:

1. Marek Capinski and Tomasz Zastawniak, Mathematics for Finance, second edition, Springer, 2004.
2. Sheldon Ross, An Elementary Introduction to Mathematical Finance, third edition, Cambridge University Press, 2011.

513016355616 Basics of Numerical Analysis

Course Objectives:

1. To execute some numerical methods for solving an algebraic or differential equation when an exact solution of the same cannot be obtained by analytical methods.
2. To test the order of convergence or stability of a method.
3. To predict the truncation error involved, in interpolating a function by a polynomial.
4. To solve ordinary differential equations with initial conditions using numerical methods.

Course Outcome:

1. Students will perform some iterative methods for solving transcendental and polynomial equations with an emphasis on the order of convergence.
2. Iterative methods for solving a system of algebraic equations, theorems pertaining to their convergence and bounds on eigenvalues will be discussed in the course.
3. Students will be able to calculate using various methods of interpolation.
4. The course incorporates solving initial value problems using numerical methods and understanding the stability of the methods together with the truncation error involved.

Pre-requisites: A course of linear algebra.

Unit-I: Transcendental and Polynomial equations (15 hours)
Newton-Raphson method, Regula Falsi and Secant methods, Ramanujan's method, Muller's method, Chebyshev method: derivation and rate of convergence. Iterative methods for polynomial equations: Birge-Vieta method and Bairstow method.
Reference: Chapter 2 of Jain, Iyengar, Jain and Chapter 2 of S.S.Sastry.

Unit-II: Numerical Linear Algebra (15 hours)
Solving a system of linear equations by Gauss elimination method, Jacobi Iteration method, Gauss-Seidel Iteration method, Convergence theorems of Jacobi and Gauss-Seidel methods.
Bounds on Eigenvalues: Gerschgorin theorem and Brauer theorem for bounds of eigenvalues of matrices, Power method for determining the largest eigenvalue of a matrix, inverse power method, singular value decomposition.
Reference: Chapter 3 of Jain, Iyengar Jain and Chapter 7 of S.S.Sastry.

Unit-III: Interpolation (15 hours)
Lagrange's interpolation, Newton's divided difference interpolation, Truncation error bound, Newton's forward and backward difference interpolation, finite difference operators, Piecewise and Spline interpolation.
Reference: Chapter 4 of Jain, Iyengar, Jain.

Unit-IV: Initial Value Problems (15 hours)
Local Truncation error, convergence and stability of numerical methods, Euler's method, Backward Euler method, Mid point method, Taylor Series method, Explicit Runge Kutta methods of second, third and fourth order.
System of Equations: explicit Runge Kutta method of fourth order.
Stability analysis of single step methods: Euler's method, Backward Euler method, explicit Runge Kutta methods.
Reference: Chapter 6 of Jain, Iyengar, Jain.

List of Practicals

1. Solving algebraic or transcendental equations by Newton-Raphson method, Regula Falsi and Secant methods, Ramanujan's method, Muller's method and Chebyshev method.

2. Solving polynomial equations by Birge-Vieta method and Bairstow method.

3. Solving a system of equations by Gauss elimination method, Jacobi iteration method and Gauss-Seidel method.

4. Finding the largest eigenvalue of a matrix by Power method and inverse power method, singular value decomposition.

5. Approximating a function by Lagrange's interpolation and Newton's forward and backward interpolation, finding a bound for the truncation error.

6. Problems based on piecewise and spline interpolation.

7. Solving an initial value problem by Euler's method, Mid point method, Taylor Series method and explicit Runge Kutta methods.

8. Solving a system of first order differential equations with initial conditions by fourth order Runge Kutta method.

Recommended Text Books:

1. M.K.Jain, S.R.K.Iyengar and R.K. Jain, Numerical Methods for Scientists and Engineers, New Age International, Sixth Edition (2014).

2. S.S.Sastry, Numerical Methods, Prentice Hall India (2013).

3. H.M.Antia, Numerical Methods for Scientists and Engineers, TMH (1991).

4. K.E.Atkinson, An introduction to Numerical Analysis, John Wiley and Sons, 1978.

513016355617: Quadratic Forms

Course Objectives:

1. To define the notion of a quadratic form and represent it via matrices.
2. To develop the concept of p-adic integers and study their properties.
3. To detect representability by a quadratic form.
4. To determine when quadratic forms represent all numbers.

Course Outcomes:

1. Students will be able to outline the basics of quadratic and symmetric forms. Moreover, they will be able to define to p-adic integers.
2. The learners will be able to summarize the notion of representability by a quadratic form along with the concepts of hyperbolic plane and isotropy.
3. The students will be able to illustrate various results related to quadratic forms over finite fields.
4. The students will be able to recall the historical theorems of Gauss and Lagrange related to triangular numbers and sums of squares.

Unit I. Basics of quadratic forms and bilinear forms (15 hours)
Revision of basics of quadratic forms, symmetric bilinear forms, discriminant of a quadratic form, radical of a quadratic form and non-degenerate quadratic form.

p-adic valuation, lifting congruences modulo higher prime powers, completions and statement of Ostrowski's theorem, ring of p-adic integers: definition and properties, field of p-adic integers, units in the ring of p-adic integers, Hensel's lemma, proof of Hensel's lemma.

Unit II. Witt's theorem and representability by a quadratic form (15 hours)
Orthogonality, radical, orthogonal direct sum, Quadratic module, non-degenerate quadratic module, orthogonal basis, isotropy, hyperbolic plane, quadratic form represents every element of the field under suitable technical conditions, existence of orthogonal basis for every quadratic module, equivalent properties for representability by a non-degenerate quadratic form.

(Ref: A Course in arithmetic, J. P. Serre, Chapter 4).

Unit III. Quadratic forms over finite fields (15 hours)
Finite fields, existence of finite fields, multiplicative group of a finite field is cyclic using Euler's ϕ function, equations over finite fields, Chevalley-Warning theorem and proof, all quadratic forms in atleast three variables over a finite fields are isotropic.

(Ref: A Course in arithmetic, J. P. Serre, Chapter 4).

Unit IV. Sums of squares and representations over integers(15 hours)
Hasse-Minkowski theorem (statement), Gauss's theorem on a positive integer to be a sum of three squares, positive definite quadratic form, equivalent conditions for positive definite quadratic form, Davenport-Cassels lemma on representations in $\mathbb{Q}$ to representations in $\mathbb{Z}$, Lagrange's four square theorem, Gauss's theorem on every positive integer being a sum of three triangular numbers.

(Ref: A Course in arithmetic, J. P. Serre, Chapter 4, Appendix).

List of Practicals

1. Basics of quadratic forms and properties.
2. Introduction to p-adic integers.
3. Non-degenerate quadratic forms and representability.
4. Examples of isotropy, hyperbolic plane.
5. Properties of finite fields.
6. Illustrations of Chevalley-Warning theorem.
7. Positive definite quadratic forms and their properties.
8. Miscellaneous examples on sums of k-th powers.

Recommended Text Books:

1. Fernando Q. Gouvea, p-adic numbers, Springer Verlag, Third Edition, Universitext.
2. Serre, J.P., A course in arithmetic, Graduate Texts in Mathematics, GTM, Volume 7.
3. Lam, T. Y, Introduction to quadratic forms over fields, Graduate Studies in Mathematics, Volume 67.
4. M. Artin, Algebra, Prentice Hall of India.

513016355618: Basic Algebraic Geometry

Course Objectives:

1. To outline the algebraic approach to understand solutions of system of equations.
2. To recognize the interplay between commutative algebra and geometry.
3. To differentiate singularity and non-singularity of varieties through algebra.
4. To convince the students of the relation between discrete valuation rings and certain topics in geometry.

Course Outcomes:

1. Students will correlate between geometry and commutative algebra after the course.
2. That topology gives comprehensive understanding of the ideal theoretic systems will get underlined.
3. The students will assemble the necessary background to understand the algebraic counterpart of the Jacobian matrix.
4. The students will correlate much better the topics of commutative algebra and algebraic geometry.

Prerequisites: Basic knowledge of algebra and commutative algebra is required.

Unit-I: Affine Variety (15 hours)
Algebraic set, affine n-space A^n over algebraically closed field k, Union and intersection of algebraic sets, Zariski topology on A^n. Irreducible space, Affine algebraic varieties, ideal I_Y of $Y \subset A^n$ of $A = k[X_1, \ldots, X_n]$, some relations between ideal I_Y and a set $Y \subset A^n$, affine coordinate ring, Hilbert's nullstellensatz.

Unit-II: Noetherian Topological space and Regular functions (15 hours)
Noetherian Topological space, Height of prime ideal, dimension of ring, Quasi-affine variety, Krull's Hauptidealsatz, one dimensional Noetherian integral domain. Regular functions, Regular function is continuous.

Unit-III: Regular functions and Morphism of Varieties (15 hours)
Morphism of Varieties, Isomorphism of Varieties, Ring of germs of regular functions on a variety, Ring $\mathcal{O}(y)$ of regular functions on a variety Y.
Function field $K(Y)$. Connection between function field, ring of germs, ring of regular functions of a variety. If B is an integral domain then B is equal to the intersection (inside its quotient field) of its localizations at all maximal ideals.
If X is any variety and Y is an affine variety then X is isomorphic to Y if and only if $A(X)$ is isomorphic to $A(Y)$ as a k-algebra. Finiteness of integral closure.

Unit-IV: Non-singular varieties (15 hours)
Regular local ring, Singularity at a point, Set $SingY$ of singular points of Y is a proper closed subset of Y.
Ideal-adic topology, m-adic topology for maximal ideal m. Completion. Properties of completion of Noetherian local ring.
Non-singular curves: Prerequisites: Integral extension, valuation ring, DVR, integrally closed ring Dedekind domain.
Any DVR of $K|_k$ is isomorphic to a point on some non singular affine curve.

List of Practicals

1. Determine algebraic sets, affine variety, Zariski Topology, irreducible sets.
2. Determine Zariski Topology, irreducible sets, connectedness and affine coordinate rings.
3. Determine height, dimension, depth altitude of ideals rings and varieties.
4. Determine One dimensional Noetherian integral domain and affine curves.
5. Determine regular functions on a variety, Ring of germs of regular functions on a variety.
6. Determine Ring of germs of regular functions on a variety, some examples of Morphism of Varieties.
7. Determine non-singular varieties, completion of rings and non-singular curves.
8. Determine completion of rings and non-singular curves.

Recommended Text Books:

1. Robin Hartshorne, *Algebraic Geometry*; Springer International.
2. C. Musili; *Algebraic Geometry for Biginners*; Hindustan Book Agency, TRIM 20.
3. Miles Reid; *Undergraduate Algebraic Geometry*; Cambridge University Press.
4. Balwant Singh, Basic commutative algebra, World Scientific.
5. Dilip P. Patil and Uwe Storch, Introduction to algebraic geometry and commutative algebra, World Scientific.

513016355619: Numerical Linear Algebra

Course Objectives:

1. To develop algorithmic approach to understand transformations.
2. To underline the unity between topics with relevant emphasis on details.
3. To discuss the technique of singular value decomposition.
4. To implement the least square method on the computer.

Course Outcomes:

1. Students will calculate the matrices for the linear transformations.
2. To develop ideas that are not normally emphasized in a standard linear algebra course.
3. To recognize which problems can be solved with linear algebra techniques.
4. To implement algorithms and demonstrate the power of computers, since calculations by hand become cumbersome for matrices of very large sizes.

Unit-I: Singular Value Decomposition(SVD) (15 hours)
Singular values, Singular vectors, SVD, Reduced SVD, Full SVD, Existence and uniqueness of SVD of $m \times n$ matrices, Difference between SVD and eigenvalue decomposition, Matrix properties via the SVD, R and python program to evaluate SVD of matrices.

Unit-II: QR Factorization (15 hours)
Projector (or Idempotent), Projection with orthonormal basis, Projection with an arbitrary basis, Reduced QR factorization, Full QR factorization. Existence and uniqueness of full (and reduced) QR factorization. Solution of $Ax = b$ by QR factorization.
Classical Gram-Schmidt (iteration or) algorithm (unstable), Modified Gram-Schmidt (iteration or) algorithm, Operation count of QR factorization.
Python and R codes for classical and modified Gram-Schmidt algorithm.

Unit-III: Householder Triangularization and Least Square Problem(LSP) (15 hours)
Discrete LSP, Orthogonal matrices, rotators, reflectors (or Householder transformations), Householder Triangularization.
Solution of LSP: Full rank case and Rank-deficient case. Geometric approach to the LSP. Discrete and continuous LSP.

Unit-IV: Eigenvalues and Eigenvectors (15 hours)
Diagonalization, Orthogonal Diagonalization, Unitary Diagonalization,, Schur Factorization, Matrix reduction to Hessenberg or Tridiagonal form. Rayleigh quotients, Inverse iteration.
Francis algorithm for computing the complete set of eigenvalues of matrix: Francis iteration of degree one and two.

List of Practicals

1. Compute reduced SVD of $m \times n$ matrix explicitly in R and Python.
2. Compute full SVD of $m \times n$ matrix explicitly in R and Python.
3. Python and R codes for classical Gram-Schmidt algorithm.
4. Python and R codes for modified Gram-Schmidt algorithm.
5. Python and R codes for Householder Triangularization.
6. Python and R codes for Least Square Problem.
7. Francis algorithm for computing the complete set of eigenvalues of matrix in R.
8. Francis algorithm for computing the complete set of eigenvalues of matrix in Python.

Recommended Text Books:

1. Lloyd N. Trefethen and David Bau; *Numerical Linear Algebra*; Siam.
2. David S. Watkins; *Fundamentals of Matrix Computations*; A Wiley-Interscience Series of Texts, Monographs, and Tracts.
3. Mike X Cohen; *Linear Algebra : Theory, Intuition, Code*; Shroff Publishers and Distributors.

513016355620: R Programming

Course Objectives:

1. To underline the benefits of Git Repository.
2. To differentiate between various kinds of graph plots.
3. To demostrate the use of loops and Apply family.
4. To explain the important concepts of statistics.

Course Outcomes:

1. Students will sketch easily different graphs and will derive initial conclusion from graphs.
2. The course illustrates the techniques to handle T-tests and non-parametric tests.
3. Students will predict using ANOVA by the end of the course.
4. The course integrates along with theory programming skill which is very much essential in today's world.

Unit-I: R Programming (15 hours)
Creating Git Repository, Basic Data types in R, Missing values and removing missing values(either in list or in vectors), Exploring packages, loading packages and learning of packages. Basic Data Management: Creating new variables; Recoding and renaming variables; Sorting, Merging, Subsetting datasets. Reading data from different files, Data Manipulation with dplyr/tidyverse.

Unit-II: Graph Plot (15 hours)
Graph with ggplot2, saving graphs.Plotting data with bar, box, and dot plots; Creating pie charts, tree maps, Histogram. Kernel Density plot, Box plot.

Unit-III: Apply Family and Exploratory Data Analysis(EDA) (15 hours)
Loops, Apply Family, Working with Dates and Time, Exploratory Data Analysis.

Unit-IV: Probability and Statistics (15 hours)
Probability, Conditional Probability, Random Variables, Expectation, Variance, Moment generating functions, Central Tendencies, Measures of central tendencies, Confidence Intervals, Hypothesis Testing, Distributions, T test and non-parametric tests. Analysis of variance and power analysis.

List of Practicals

1. Manage R workspace; Exploring, loading and learning of packages.
2. Understanding data sets; R data structures; Crating Data Frames; Data input(import data); Annotating datasets.
3. Basic Data Management: Creating new variables; Recoding and renaming variables; Sorting, Merging, Subsetting datasets.
4. Use of dplyr to manipulate data frames;
5. Creating a graph with ggplot2, saving graphs.
6. Functions in R for manipulating data: Mathematical, Statistical and probability functions. Other useful functions.
7. Plotting data with bar, box, and dot plots; Creating pie charts, tree maps, Histogram.
8. Kernel Density plot, Box plot.
9. Descriptive Statistics.
10. Frequency and contingency tables.
11. T-tests. Non-parametric tests of group differences.
12. Anova.

Recommended Text Books:

1. Laura M. Chihara and Tim C. Hesterberg; *Mathematical Statistics with Resampling and R*; Wiley Publisher.
2. Robert I. Kabacoff; *R in Action*, *Data Analysis and Graphics with R and Tidyverse*; Manning publication.
3. Nina Zumel and John Mount; *Practical Data Science with R*; Manning publication.
4. Hadley Wickham, Mine Çetinkaya-Rundel and Garrett Grolemund; *R for Data Science*; O'Reilly publication.

Semester-IV

5130164561: Algebra IV

Course Objectives:

1. To recall the fascinating history of solving polynomial equations and its relation to group theory.
2. To identify normal extensions and separable extensions.
3. To evaluate the Galois group in specific cases.
4. To recognize cyclotomic polynomials and develop their relation with the roots of unity.

Course Outcomes:

1. Students will be able to test if the given extensions are algebraic and determine their properties.
2. Learners will compute the splitting fields and their degrees. Normal extension is defined and its equivalent properties are discussed.
3. Interpretation of finite fields as splitting fields and the notion of algebraic closure will be explained in detail by the students at the end of course.
4. Galois extensions can be classified by students and the fundamental theorem of Galois theory will be illustrated via examples.

Unit I. Algebraic Extensions (15 hours)

Prime subfield of a field, definition of field extension K/F, algebraic elements, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element.

Algebraic extensions, Finite extensions, degree of an algebraic element, degree of a field extension. If α is algebraic over the filed F and $m_\alpha(x)$ is the minimum polynomial of α over F, then $F(\alpha)$ is isomorphic to $F[X]/(m_\alpha(x))$. If $F \subseteq K \subseteq L$ are fields, then $[L : F] = [L : K][K : F]$. If K/F is a field extension, then the collection of all elements of K which are algebraic over F is a subfield of K. If L/K, K/F are algebraic extensions, then so is L/F. Composite filed K_1K_2 of two subfields of a field and examples. (Ref:D.S. Dummit and R.M. Foote, Abstract Algebra)

Classical Straight-edge and Compass constructions: definition of Constructible points, lines, circles by Straight-edge and Compass starting with $(0, 0)$ and $(1, 0)$, definition of constructible real numbers. If $a \in \mathbb{R}$ is constructible, then a is an algebraic number and its degree over $\mathbb{Q}$ is a power of 2. $\cos 20^0$ is not a constructible number. The regular 7-gon is not constructible. The regular 17-gon is constructible. The Constructible numbers form a subfield of $\mathbb{R}$. If $a > 0$ is constructible, then so is $\sqrt{a}$. (Ref:M. Artin, Algebra, Prentice Hall of India).

Impossibility of the classical Greek problems: 1) Doubling a Cube, 2) Trisecting an Angle, 3) Squaring the Circle is possible. (Ref:D. S. Dummit and R.M. Foote,Abstract Algebra).

Unit II. Normal and Separable Extensions (15 hours)

Splitting field for a set of polynomials, normal extension, examples such of splitting fields of $x^p - 1$ (p prime), uniqueness of splitting fields, existence and uniqueness of finite fields. Algebraic closure of a field, existence of algebraic closure.

Separable elements, Separable extensions. In characteristic 0, all extensions are separable. Frobenius automorphism of a finite field. Every irreducible polynomial over a finite field is separable. Primitive element theorem.

(Reference for Unit II: D. S. Dummit and R. M. Foote, Abstract Algebra).

Unit III. Galois Theory (15 hours)

Galois group $G(K/F)$ of a field extension K/F, Galois extensions, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory, Cyclotomic field $Q(\zeta_n)$ (splitting field of $x^n - 1$ over $\mathbb{Q}$), cyclotomic polynomial, degree of Cyclotomic field $Q(\zeta_n)$. (Reference: D. S. Dummit and R.M. Foote, Abstract Algebra)

Unit IV. Applications (15 hours)

Galois group of the cyclotomic field, Galois group for an irreducible cubic polynomial, Galois group for an irreducible quartic polynomial. (Ref: M. Artin, Algebra, Prentice Hall of India).

Solvability by radicals in terms of Galois group and Abel's theorem on the insolvability of a general quintic. (Ref: D. S. Dummit and R. M. Foote, Abstract Algebra)

List of Practicals

1. Algebraic extensions and related properties.
2. Constructible numbers.
3. Splitting field, normal extensions.
4. Separability and related properties.
5. Galois groups and fixed fields.
6. Cyclotomic polynomials and fields.
7. Galois groups of cubic and quadrtic polynomials.
8. Insovability of quintic polynomials.

Recommended Text Books:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley and Sons.
2. M. Artin, Algebra, Prentice Hall of India, 2011.
3. S. Lang, Algebra, Springer Verlag, 2004
4. N. Jacobson, Basic Algebra, Dover, 1985.

5130164562: Fourier Analysis

Course Objectives:

1. To study conditions under which a function has a Fourier series expansion and its properties.
2. Development of the Dirichlet's kernel, the Fejer kernel and the Poisson kernel and study their applications.
3. To analyze the convergence of the Fourier series.
4. To study the Dirichlet's problem and development of heat kernel as an application of the Fourier series.

Course Outcome: On completion of this course the learner will be able to

1. Generate the Fourier series expansion of a periodic function and analyze their convergence.
2. Grasp the properties of the Dirichlet kernel, Fejer kernel, Poisson kernel and the concept of a good kernel.
3. Obtain a Fourier series solution of the Dirichlet problem and the heat equation.

Unit-I: Fourier Series (15 hours)
The vibrating string problem, one dimensional heat conduction problem and its solution by the separation of variables. Dirichlet's conditions, Definition of the Fourier series of a periodic function, Bessel's inequality for a 2π periodic Riemann integrable function, Derivatives, Integrals and Uniform Convergence properties, Fourier series on intervals, Even & odd extensions, Fourier series of a periodic function of an arbitrary period, Uniqueness theorem.

(Reference for Unit I: Unit 1 of E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003. Sections 2.1, 2.2, 2.3, 2.4 of the G.B. Folland, Fourier Analysis and its Applications, American Mathematical Society, Indian Edition 2010.)

Unit-II: Dirichlet's Kernel and applications (15 hours)
Dirichlet kernel, Convergence theorem for the Fourier series of a periodic and piecewise smooth function, Fourier coefficients of integrable and square integrable periodic functions, The Riemann-Lebesque lemma and its converse, Bessel's inequality for a periodic functions, Dirichlet's theorem, Concept of Good kernels, Dirichlet's kernel is not good kernel.

(Reference for Unit II: Sections 2.2 of the G.B. Folland, Fourier Analysis and its Applications, American Mathematical Society, Indian Edition 2010. Sections 13A, 13B, 13C of R. Beals, Analysis An Introduction, Cambridge University Press, 2004 and Section 4 of unit 2 of E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003, for converse of Riemann-Lebesgue lemma refer section 5.14 of W. Rudin, Real and Complex Analysis, Tata McGraw Hill)

Unit-III: Fejer's Kernel and applications (15 hours)
Cesaro summability, Cesaro mean and Cesaro sum of the Fourier series, Fejer's Kernel, Fejer's kernel is a good kernel, Fejer's Theorem, Weierstrass approximation theorem as an application, Parseval's identity. Convergence of Fourier series of an L^2 periodic function w.r.t the L^2-norm, Riesz-Fischer theorem on Unitary isomorphism from $L^2[-\pi, \pi]$ onto the sequence space l^2 of square summable complex sequences.

(Reference for Unit III: Sections 13D, 13E, 13F of R. Beals, Analysis An Introduction, Cambridge University Press, 2004, Section 5.1 & 5.2 of unit 2 of E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003 and Sections 2.2, 3.2 of E. M. Stein and R. Shakarchi, Real Analysis an Introduction, New age International).

Unit-IV: Poisson kernel and applications (15 hours)
Abel summability, Abel sum of the Fourier series, The Poisson kernel, The Poisson kernel is a good kernel, Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The solution of Dirichlet problem for the unit disc. The Poisson integral, Applications of Fourier series to heat equation on the circle and development of heat kernel.

(Reference for Unit IV: Section 2 of unit 1, Section 5.3 & 5.4 of unit 2 and section 1 & 4 of unit 4 of E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003.)

List of Practicals

1. Examples based on the one dimensional wave equation subjected to arbitrary initial displacement.

2. Examples based on the one dimensional heat equation subjected to arbitrary initial temperature.

3. Find the Fourier series of real and complex valued periodic functions of the period 2π.

4. Find the Fourier series of real and complex valued periodic functions of the arbitrary period.

5. Examples based on even and odd extension of the function and its Fourier series.

6. Examples based on Unitary isomorphism.
7. Examples based on the Dirichlet's problem in the unit disc subjected to arbitrary temperature on the boundary of the disc.
8. Examples based on the Poisson kerenel.
9. Examples based on the Cesaro summability and Abel summability.

Recommended Text Books:

1. G.B. Folland, Fourier Analysis and its Applications, American Mathematical Society, Indian Edition 2010.
2. E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003.
3. E. M. Stein and R. Shakarchi, Real Analysis an Introduction, New age International
4. R. Beals, Analysis An Introduction, Cambridge University Press, 2004

5130164563: Functional Anaysis

Course Objectives:

1. This course aims to introduce the concept of Hilbert space, Banach space and their properties.

2. To study the concept of normed and inner product spaces, bounded linear transformation and orthogonal decomposition of Hilbert space.

3. To understand the concept of dual spaces and their properties.

4. Make to understand the concept of separable and reflexive spaces. To study some properties of normed spaces such as ℓ^p and $L^P[a,b]$.

5. To under stand the concepts of fundamental theorems such as Open mapping and closed graph theorem. Also to understand the concept of uniform boundedness principle.

Course Outcome:

1. Students will learn Hilbert and Banach spaces.

2. Students will be able to understand the concept of bounded linear transformations on normed space.

3. Students will learned the spaces such as ℓ^p and $L^p[a,b]$ and to determine their dual spaces.

4. Student will understand the open mapping and closed graph theorem and their applications.

5. Students will understand the Riesz lemma, Riesz Representation theorem and their applications.

Unit-I: Normed Linear Spaces (15 hours)
Normed Linear spaces, Banach spaces. Quotient spaces, Examples of Normed linear spaces, Arzela-Ascoli theorem, Holder's, Minkowski's and Cauchy-Schwarz inequalities. Examples of Banach spaces such as ℓ^p and $L^p[a,b]$ spaces for $1 \leq p \leq \infty$. Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and its application to a normed linear spaces.

Unit-II: Bounded Linear Transformations (15 hours)
Bounded linear transformations, Equivalent characterizations. $\mathcal{B}(X,Y)$ a space of bounded linear transformation, completeness of $\mathcal{B}(X,Y)$. Dual space of a normed linear space, Dual space of ℓ^p and $L^p[a,b]$ $(1 \leq p \leq \infty)$. Separable spaces and their examples. Riesz Representation theorem.

Unit-III: Hilbert Spaces (15 hours)
Inner product and its properties, inner product spaces, complete inner product spaces, Hilbert spaces, ℓ^2 and $L^2[-\pi, \pi]$ spaces. Norm induced by inner product and vice-versa. Orthogonal sets and orthogonal decomposition of Hilbert space. Equivalence of complete orthonormal set and maximal orthonormal set. *Gram-Schmidt orthogonalization process.* Bessel's inequality and Parseval's identity. Existence of a maximal orthonormal set, separability of Hilbert space. Riesz Representation theorem for Hilbert spaces.

Unit-IV: Reflexive spaces and Fundamental Theorems (15 hours)
Bidual space, canonical mapping from space X into X''. Reflexive spaces and their examples. Reflexivity of Hilbert space, ℓ^1 is not reflexive, relation between reflexivity and separability. Hahn-Banach theorem for normed spaces and its applications. Open mapping and Closed graph theorem, Uniform boundedness principle and its application.

List of Practicals:

1. Verification of norm function and computation of norm, To test the equivalence of given norms norms.
2. Verify the completeness of given norm and example of Banach spaces.
3. Examples of bounded and unbounded transformations.
4. Example based on Separable and dual spaces.
5. Computation of inner product. Completeness of inner product space
6. Example of Hilbert space. Construction of orthonormal set.
7. Examples of reflexive spaces
8. Examples based on open mapping theorem and closed graph theorem.
9. Problem based on Uniform bounded principle.

Recommended Text Books:

1. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer International Edition, 1996.
2. E. Keryszig, *Introductory Functional Analysis with Applications*, Wiely India, 1978.
3. B. V. Limaye, *Functional Analysis*, New Age International, 1996.
4. M. T. Nair, *Functional Analysis*, Prentice Hall, India
5. H. L. Royden, *Real Analysis*, Pearson, 4th edition, 2017.

6. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

7. Peter D. Lax, *Functional Analysis*, Wiley.

5130164564: Research Project

In order to maintain its quality and purpose of research projects, the following guidelines has to be follow

1. Research project batch will consist of minimum 3 and maximum 8 students.
2. A research project can consist of different research topices depending on the students and research project supervisor.
3. The students will be evaluated on their individual performance and their contribution to the research project.
4. Students has to do detailed literature review on their respective research topic.
5. It is mandatory to spend at least two hours a week on e-learning resources as practical work during research project.
6. Use of soft computing tools/ mathematical softwares viz Matlab, Mathematica, MathCAD etc. will be highly appreciated.
7. For semester IV, a student may be allowed to continue with the research topic done by her/him in Semester III. However the material submitted for evaluation in Semester 3 cannot be included for evaluation in Semester IV.
8. A student has option of choosing a research topic and supervisor different from the one in Semester III.

513016456511: Lie Algebras

Course Objectives:

1. To define Lie algebras.
2. To illustrate basic concepts in Lie algebras like ideals, homomorphisms, subalgebras, derived algebras.
3. To discuss Engel's theorem.
4. To develop the notion of root systems.

Course Outcomes:

1. Students will recognize Lie algebras and their many examples.
2. They will be able to calculate the matrix of the adjoint representation.
3. The students will use Killing form and develop Cartan's criterion for solvability of Lie algebras.
4. The students will be able to test if a given set of vectors forms a root system and determine certain Weyl groups.

Unit I. Basics of Lie algebras (15 hours)
Definition of Lie algebras, examples, Jacobi identity, classical algebras and their examples, Lie algebras of upper triangular matrices, strictly upper triangular matrices, diagonal matrices. Subalgebras of the general linear algebra, ideals and homomorphisms, derived algebra.

Unit II. Simple Lie algebras and adjoint representation (15 hours)
Simple Lie algebras, examples, definition of derivation and examples, adjoint representation of a Lie algebra, automorphisms of a Lie algebra, inner automorphisms, ad-nilpotent and conjugation by exponential.

Unit III. Further types of Lie algebra (15 hours)
Solvable Lie algebras, nilpotent Lie algebras, examples from general linear Lie algebra, Engel's theorem, adjoint of nilpotent endomorphism, existence of common eigenvector for Lie algebra of nilpotent endomorphisms. Existence of Jordan-Chevalley decomposition, semisimple and nilpotent elements.

Unit IV. Introduction to root systems (15 hours)
Cartan's criterion for solvability, Killing form, characterization of semisimple Lie algebras using non-degeneracy of Killing form, Definition of Root system, reflections, hyperplanes, Weyl group, conjugation action of the automorphism group on the Weyl group, pairs of roots and possible angles, Computation of some root systems of rank two, computation of some Weyl groups.

List of Practicals

1. Examples of Lie algebras
2. Subalgebras, ideals, homomorphisms in Lie algebras.
3. Simple Lie algebras.
4. Adjoint representation.
5. Solvable Lie algebras and semisimple elements.
6. Nilpotent Lie algebras and nilpotent elements.
7. Root systems of rank two: classification.
8. Simple computations of Weyl groups.

Recommended Text Books:

1. James E. Humphreys, Introduction to Lie Algebras and Representation Theory, Graduate Text in Mathematics, Volume 9.
2. S. Lang, Algebra, Springer Verlag, 2004
3. N. Jacobson, Basic Algebra, Dover, 1985.

513016456512: Representation Theory of Finite Groups

Course Objectives:

1. To define a representation and list examples.
2. To discuss characters and orthogonality relations.
3. To demonstrate the role of Frobenius reciprocity.
4. To associate character tables to groups.

Course Outcomes:

1. Students will recognize what is a representation and their examples. Also they will give original examples of invariant vectors and invariant subspaces.
2. They will compute the characters of various representations.
3. The students will be able to check the orthogonality relations for characters.
4. The Frobenius reciprocity law will help students to calculate further examples of character tables.

Unit I. Basic Definitions (15 hours)
Definition, trivial representation, sign representation, regular representation, faithful representatiion, matrix representation, examples, dimension of representation, character of a representation, character is constant on conjugacy class, G-invariant subspaces, G-invariant vector, examples, definition of irreducible representation.
(Reference: Artin, Algebra).

Unit II. Maschke's theorem (15 hours)
Equivalence of representations, relation between character of equivalent representations, reducible representation, standard representation, examples of representations, Maschke's theorem, Unitary representation, existence of G-invariant, positive definite Hermitian form on V for a representation ρ of $GL(V)$. Corollaries of Maschke's theorem.
(Reference for Unit II: Artin, Algebra).

Unit III. Characters and orthogonality relations (15 hours)
Definition of character, examples of characters, properties of characters like relation to dimension, proof of orthogonality relations for characters, character tables, examples of character tables for small groups.

Unit IV. Frobenius reciprocity and examples of character tables (15 hours)
Induced representation, restriction of a representation, statement of Frobenius reciprocity, proof of the statement of Frobenius reciprocity law, further examples of character tables.

List of Practicals

1. Representation definition and examples.
2. Invariant vectors and subspaces.
3. Averaging trick and unitary representations.
4. Examples of representations.
5. Character tables of groups of small order.
6. Orthogonality relations and characters.
7. Frobenius reciprocity.
8. Further examples of character tables.

Recommended Text Books:

1. Gordon James and Martin Liebeck, Representations and characters of groups, Second edition, Cambridge.
2. M. Artin, Algebra, Prentice Hall of India, 2011.
3. Serre, Linear Representations of finite groups, Graduate Texts in Mathematics, GTM, Volume 42.
4. Peter Webb, A Course in Finite Group Representation Theory,

513016456513: Special Functions

Course Objectives:

1. To study the hypergeometric equation and its solutions.
2. To study the properties of hypergeometric functions and its generating functions.
3. To study the solution of Legendre's differential equations and evaluation of Legendre's polynomials.
4. To study Bessel's functions and its applications.
5. To solve Hermite equations and its solutions

Course Outcome:

1. Students will learn hypergeometric functions and their applications .
2. Student will able to solve special kind of second order Legendre's differential equations.
3. To be understand first and second kind of Bessel's functions, generating functions and its orthogonal properties.
4. Students will be able to understand the Hermite polynomials and its properties. Use it to solve some specific type of second order differential equations.

Unit-I: Hypergeometric Functions (15 hours)
[Revision of ordinary points, regular singular points, points at infinity and method of series solutions-only for revision]. The hypergeometric series, integral formula for hypergeometric series, Gauss theorem, Vander-Monde's theorem, hypergeometric equation, relation of contiguity, the confluent hypergeometric function.

Unit-II: Legendre's Functions (15 hours)
Legendre polynomials, recurrence relations, Morphy and Rodrigues formula, generating function for Legendre polynomials, orthogonality property, Legendre series, relation between Legendre polynomial and its derivatives, associate Legendre functions, Neumann's formula.

Unit-III: Bessel's Functions (15 hours)
Bessel's equation and its solution, Bessel's functions of first and second kind, generating function for Bessel functions, integral representation, recurrence relations, Hankel function, modified Bessel functions, orthogonality property, Bessel's series.

Unit-IV: Hermite Polynomials (15 hours)
Hermite equation and its solution, generating function for Hermite polynomials, explicit expression for, and special values of the Hermite polynomials, orthogonality property of Hermite polynomial, relation between Hermite polynomials and its derivatives, Weber-Hermite functions.

List of Practicals

1. Problems based on hypergeometric functions and derivatives.
2. Problems based on integral representation of hypergeometric functions.
3. Problems based on Confluent hypergeometric function.
4. Solution of Legendre differential equations
5. Problems based on properties of Legendre's polynomials.
6. Solution of Bessel's differential equations
7. Problems based on derivatives of Bessel's functions and modified Bessel functions.
8. Problems based on Hermite equation.
9. Problems based on Hermite polynomials and Weber-Hermite functions.

Recommended Text Books:

1. W. W. Bell, *Special Functions for Scientists and Engineers*, D. Van Nostrand Company Inc., Princeton, New Jersey, 1963.
2. Ian N. Sneddon, *Special Functions of Mathematical Physics and Chemistry*, Oliver and Boyd Ltd., 1966.
3. George E. Andrews, Richard Askey and Ranjan Roy, *Special Functions*, Cambridge University Press, 2010.
4. N. N. Lebedev, (translated by Richard A. Silverman), *Special Functions and Their Applications*, Prentice-Hall Inc., London, 1965.
5. Rainville E. D., *Special Functions*, the Macmillan Co., New York, 1960.

513016456514: Calculus on Manifolds

Course Objectives:

1. To introduce the concept of multilinear function and its properties.
2. To understand and apply the concepts of differential form, pullback form, closed and exact form.
3. To understand and apply the concept of exterior derivatives.
4. To study the Green's theorem, divergence theroem and Stokes theorem and their applications.

Course Outcomes: On completion of this course the learner will be able to

1. Grasp the concept of tensor, alternating tensor, wedge product and differential forms and compute them.
2. Analyze fields and forms on manifolds.
3. Understand and apply the Classical theorems: Stoke's theorem, Green's theorem, Gauss divergence theorem.

Prerequisites: Differential Geometry.

Unit I: Multilinear Algebra (15 hours)
Multilinear map on a finite dimensional vector space V over $\mathbb{R}$ and $k-$tensors on V, the collection $\Im^k(V)$ or $\otimes^k(V^*)$ of all $k-$tensors on V, tensor product $S \otimes T$ of $S \in \Im^k(V)$ and $T \in \Im^l(V)$, alternating tensor and the collection $\Lambda^k(V)^*)$ of $k-$tensors on V, the exterior product or wedge product, basis of $\Lambda^k(V)^*)$, orientation of finite dimensional vector space V over $\mathbb{R}$.
(References for Unit I: PP $75 - 84$ in Chapter IV of M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc.)

Unit II: Differential Forms (15 hours)
Differential forms or $k-$forms on $\mathbb{R}^n$, wedge product of $k-$forms ω and $l-$forms η, the exterior derivative and its properties, Pullback forms and its properties, closed and exact forms, Poincare's lemma.
(Reference for Unit II: PP $86 - 97$ in Chapter IV of M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc.)

Unit III: Basics of Submanifolds of $\mathbb{R}^n$ (15 hours)
Submanifolds of $\mathbb{R}^n$, submanifolds of $\mathbb{R}^n$ with boundary, smooth functions defined on submanifolds of $\mathbb{R}^n$, Tangent vector and tangent space of submanifolds of $\mathbb{R}^n$.

$p-$forms and differential $p-$forms on a submanifolds of $\mathbb{R}^n$, exterior derivative $d\omega$ of any differential $p-$forms on a submanifolds of $\mathbb{R}^n$, orientable submanifolds of $\mathbb{R}^n$ and oriented submanifolds of $\mathbb{R}^n$, Orientation preserving map, Vector fields on submanifolds of $\mathbb{R}^n$, outward unit normal on the boundary of a submanifolds of $\mathbb{R}^n$ with nonempty boundary, induced orientation of the boundary of an oriented submanifolds of $\mathbb{R}^n$ with nonempty boundary.
(Reference for Unit III: PP $109-122$ in Chapter V of M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc.)

Unit IV: Stoke's Theorem (15 hours)
Integral $\int_[0,1]^k\omega$ of a $k-$forms on cube $[0,1]^k$, Integral $\int_c \omega$ of a $k-$forms on an open subset A of $\mathbb{R}^k$ where c is a singular $k-$ cube in A, Stoke's Theorem for $k-$ cube, if ω is $k-1$form on an open subset A of $\mathbb{R}^k$ and c is a singular $k-$ cube in A then $\int_c \omega = \int_{\partial c} \omega$.
(Reference for Unit IV: PP $100-108$ in Chapter IV of M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc.)

Integration of a differentiable $k-$forms on oriented k dimensional submanifolds M of $\mathbb{R}^n$, Change of variables theorem, if $c_1, c_2 : [0,1]^k \longrightarrow M$ are two Orientation preserving maps in M and ω is any $k-$forms on M such that $\omega = 0$ outside of $c_1([0,1]^k) \cap c_2([0,1]^k)$ then $\int_{c_1} \omega = \int_{c_2} \omega$, Stokes' theorem for submanifolds of $\mathbb{R}^k$, Volume element, Integration of functions on a submanifold of $\mathbb{R}^k$, Classical theorems: Green's theorem, Divergence theorem of Gauss, Green's identities.
(Reference for Unit IV: PP $122-137$ in Chapter V of M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc.)

List of Practicals

1. Examples based on the tensor production and wedge product.
2. Examples based on the alternating tensor.
3. Examples based on the exterior derivatives.
4. Examples based on the closed and exact forms.
5. Examples based on the manifolds in $\mathbb{R}^n$ without boundary.
6. Examples based on the manifolds in $\mathbb{R}^n$ with boundary.
7. Examples based on Green's theorem.
8. Examples based on divergence theorem.
9. Examples based on Stoke's theorem.

Recommended Text Books:

1. M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc.
2. V. Guillemin and A. Pollack, Differential Topology, AMS Chelsea Publishing, 2010.
3. J. Munkers, Analysis on Manifolds, Addision Wesley.
4. A. Browder, Mathematical Analysis, Springer International Edition.

513016456515: Calculus of Variations

Course Objectives:

1. To understand the concept of variational methods in Euclidean space and Banach space
2. To understand the concept of convex sets, convex functions and their importance in optimization problems
3. To learn techniques to find minimizers for given problem
4. To learn techniques to find saddle point solutions to given problem

Course Outcome:

1. Analyze given problem and apply suitable variational method to solve it.
2. Solve convex optimization problems.
3. Find minimizers applying the given conditions for solving problems in finite dimension and finding solutions of PDE.
4. Find saddle point solution applying the Mountain pass theorem in finite dimension and in Banach spaces to solve PDEs.

Prerequisites: Several variable Calculus, Basic course in PDE, Functional Analysis.

Unit I: Finite Dimensional Case (15 hours)
Review of finding critical points in $\mathbb{R}^n$, second derivative test. Lower semicontinuity, Lagrange multiplier theorem, Notions of differentiability, conditions of local optimality, Danskin's theorem, parametric monotonicity of optimizers, Ekeland variational principle, Mountain pass theorem.

Reference: Chapter 2 of "Elementary convexity with optimization" Vivek S. Borkar and K.S.Mallikarjuna Rao, Trim series Hindustan book agency

Unit II: Convex Optimization (15 hours)
Convex sets, minimum distance problem, separation theorems, Brouwer fixed point theorem. Convex functions- continuity, differentiability, approximation. Legendre and Fenchel duality, Lagrange multiplier rule

Reference: Chapter 3, 4, 5 of "Elementary convexity with optimization" Vivek S. Borkar and K.S.Mallikarjuna Rao, Trim series Hindustan book agency

Unit III: Critical points in Banach Space (15 hours)
Coercivity, Existence of Minimizers, Palais-Smale condition, Deformation lemma, Critical points of mountain pass type

Reference: Section 1, 2, 3 and 9 of Chapter 2 of Variational methods by Michael Struwe.

Unit IV: Solving PDEs using variational methods (15 hours)
Application to game theory: Min-Max theorem, existence of Nash equilibria, Application to finding solutions of PDE: Existence of minimizers, weak solutions of Euler Lagrange equations, nonlinear eigenvalue problem, application of mountain pass theorem for existence of solution to semilinear elliptic problem.

Reference: Chapter 5, Section 5.6 of "Elementary convexity with optimization" Vivek S. Borkar and K.S.Mallikarjuna Rao, Trim series Hindustan book agency and Section 8.2, 8.4 and 8.5 Chapter 8 of Partial Differential Equations, L.C. Evans Graduate studies in Mathematics, Vol. 19 AMS.

List of Practicals

1. Finding and classifying critical points, Lagrange multiplier rule
2. Lower semicontinuity, Ekeland variational principle
3. Local optimizers, Mountain Pass theorem, Applications of theorems in Unit 1
4. Convex sets, convex functions and their properties
5. Approximation, Applications of theorems in Unit 2
6. Verifying coercivity, Palais-Smale condition
7. Applications of theorems in Unit 3
8. Solving specific PDES using variational methods

Recommended Text Books:

1. Lawrence C. Evans, Partial Differential Equation, Second edition, American Mathematical Society, 2010.
2. Vivek S. Borkar and K.S.Mallikarjuna Rao, Elementary convexity with optimization, Trim series Hindustan book agency, Springer, 2023.
3. Michael Struwe, Variational methods, Springer, 2008.

513016456516: Boundary Value Problems

Course Objectives:

1. The main of this course is to study the real life problems in the context of initial and boundary value problems in the bounded and unbounded domains.
2. To study the Dirichlet and Neumann problems and their mahematical solution using various integral transforms.
3. To learn the development and properties of Green's function and its role in the fundamental solution of boundary value problems.
4. To study the Sturm-Liouville boundary value problem and its application.

Course Outcome: After successful completion of this course, students will learn

1. the mathematical modelling of real life problems in the science and engineering.
2. the mathematical solution and its physical interpretation of real life problems in the science and engineering using various integral transforms.
3. to distinguish parabolic, hyperbolic and elliptic partial differential equations and their physical interpretation.
4. identify study the Sturm-Liouville boundary value problem and application of their mathematical properties.

Prerequisites: Partial Differential Equations and Integral Transforms.

Unit I: Boundary Value Problems in Bounded Domain (15 hours)
Basic concepts and definitions, the linear superposition principle, types of boundary conditions, well posed boundary value problem, classical linear models, the second order linear PDE and its classification, Method of characteristics, canonical form, the Cauchy problem, Boundary Value Problems in Bounded Domain, Method of Separation of Variables, Tranverse vibration of a string, one and two dimensional diffusion equation in bounded domain, Dirichlet's problem for a circle.

Unit II: Boundary Value Problems in unbounded Domain (15 hours)
Fourier transform method, convolution theorem, the Cauchy problem for wave and diffusion equation in unbounded domain, Dirichlet's problem in the half plane, Neumann's problem in the half plane, the Dirichlet problem for the three dimensional Laplace equation, the Cauchy problem for the two dimensional wave equation. Multiple Fourier transforms, Dirichlet problem for three dimensional Laplace equation, Cauchy problem for the two dimensional wave equation, Laplace transform method, Hankel transform method.

Unit III: Green's Function and Boundary Value Problems (15 hours)
Green's function: definition and properties, Green's function and fundamental solution, Green's function for one and two dimensional diffusion equation, Green's function for three dimensional poisson equation, two and three dimensional Helmholtz equation, Wave equation.

Unit IV: Sturm-Liouville Boundary Value Problems (15 hours)
Sturm-Liouville problem, Cauchy-Euler equation, Sturm-Liouville problem involving Legendres and Bessel equation, Lagranges identity, Abel's formula, properties of eigenvalue of Sturm-Liouville system, orthogonal and orthonormal eigenfunction, Paresval's relation, solution of Sturm-Liouville problem associated with the wave equation and diffusion equation, Green's function for SL system and its properties, Bilinear expansion of Green's function, Higher dimensional equations, Uniquness of solution, maximum-minimum principle.

List of Practicals

1. The linearized Korteweg-de Vries equation
2. The Schrodinger equation in quantam mechanics
3. Inhomogeneous Cauchy problem for wave equation
4. Heat conduction equation in a finite and semi-infinite medium
5. Wave equation for the tranverse vibration in a sem infinite string
6. The Cauchy-Poisson wave problem in fluid dynamics
7. Steady temperature distribution in a semi-infinte solid with steady heat source
8. Axisymmetric accoustic radiation problem
9. Axisymmetric biharmonic equation, axisymmetric Cauchy-Poisson water wave problem
10. Tranverse vibration of a thin elastic circular membrane

Recommended Text Books:

1. Lokenath Debnath, Nonlinear Partial Differential Equations for Scientits and Engineers, Second edition, University of Texas, 2004.
2. Lawrence C. Evans, Partial Differential Equation, Second edition, American Mathematical Society, 2010.
3. Thomas Alazard and Claude Zuily, Tools and Problems in Partial Differential Equations, Springer, 2020

513016456517: Numerical Methods for Partial Differential Equations

Course Objectives:

1. The course aims at empowering the students with finite difference methods to find the approximate solution of an initial or boundary value problem when the exact solution is not possible to attain.

2. Errors creep into the solution when derivatives are approximated by differences and hence it is envisaged that the students understand the convergence and stability aspects of the numerical scheme.

3. The concept of well-posed problem and Lax equivalence theorem have to be understood and hence forms a part of this course.

4. The CFL condition for a hyperbolic equation and the notion of errors for all forms of partial differential equations is an important component of this course.

Course Outcome:

1. The students will be able to understand the concept of well-posed problems, truncation error, convergence and stability of a numerical scheme.

2. They can solve a parabolic equation by explicit and implicit schemes with a prior knowledge of the advantages and disadvantages of each.

3. The course develops various methods for solving a hyperbolic equation together with initial and boundary conditions.

4. The CFL condition, Lax equivalence theorem and various iterative methods for solving a system of equations arising from an elliptic equation are some other take away contents of this course.

Prerequisites: Partial Differential Equations, One course of Numerical Analysis

Unit-I: Finite difference method (15 hours)
Characterization of a two-dimensional partial differential equation, criteria for a well-posed problem, Laplace' equation with Dirichlet, Neumann or mixed boundary conditions, heat conduction problem, Cauchy problem for the one-dimensional wave equation, discretization of a domain, representing derivatives by forward, backward or central differences, convergence, stability, von-Neumann stability.
Reference: Chapters 1 and 2, P.Niyogi, S.K.Chakrabarty amd M.K.Laha, An itroduction to Computational Fluid Mechanics, Part=I, Pearson Education India, 2006.

Unit-II: Parabolic Equations (15 hours)
Transformation to non-dimensional form, an explicit method of solution, Crank-Nicolson implicit method, a weighted average approximation, local truncation error and consistency, convergence and stability, Vector and matrix norms, a necessary and sufficient condition for stability (constant coefficients), Stabilty by Fourier series method, Lax equivalence theorem(Statement only).
Reference: Chapter 2, G.D.Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods.

Unit-III: Hyperbolic Equations (15 hours)
Characteristics, System of conservation laws, the CFL condition, Error analysis of the upwind scheme, the Lax-Wendroff scheme, the Lax-Wendroff method for conservation laws, finite volume schemes, leap-frog scheme.
Reference: Chapter 4, Morton and Mayers, Numerical Solution of partial Differential equations.

Unit-IV: Elliptic Equations (15 hours)
The torsion problem, Derivative boundary conditions in a heat-conduction problem, Formulae for derivatives near a curved boundary when using a square mesh, improvement of accuracy of solutions, analysis of the discretization error in case of Poisson's equation, solution of large systems of algebraic equations by successive over-relaxation method, a necessary and sufficient condition for its convergence.
Reference: Chapter 5, G.D.Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods.

List of Practicals

1. Well-posed problem, knowledge of Dirichlet problem, Cauchy problem, heat conduction problem.

2. Convergence and stability of a numerical scheme.

3. Solving one-dimensional heat equation by explicit scheme and Crank-Nicolson method subject to certain initial and boundary conditions.

4. Finding the truncation error and condition for stability in either case.

5. Solving a linear advection equation by the method of characteristics and analysing a system of conservation laws.

6. The CFL condition, the upwind scheme, Lax-Wendroff scheme, the Leap-frog scheme for a hyperbolic equation.

7. Solving the Laplace's equation by finite difference method using SOR scheme subsequently.

8. Finding the necessary and sufficient condition for the convergence of the iterative methods.

Recommended Text Books:

1. G.D.Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, Oxford University Press (2010).
2. K.W.Morton and D.F.Mayers, Numerical Solution of partial Differential equations, Cambridge University Press (2005).
3. P. Niyogi, S.K.Chakrabarty amd M.K.Laha, An itroduction to Computational Fluid Mechanics, Part=I, Pearson Education India, 2006.
4. R.Mitchell and S.D.F.Griffiths, The finite difference methods in partial differential equations, Wiley and Sons, NY, 1980.

513016456518: Stochastic Calculus for Finance

Course Objectives:

1. To understand relation between concepts of Probability and Analysis
2. To analyze the properties of Brownian motion
3. To understand the concept of Itô integral and its properties
4. To learn the concept of strong and weak solution of a Stochastic differential equation
5. To study conditions under which the Stochastic differential equations can be solved

Course Outcome: On completion of the course, the learner will be able to

1. relate concepts of probability, calculus, measure theory, ODE, PDE with those of practical finance
2. apply Itô integral and its properties
3. apply Itô's formula, chain rule and integration by parts for Itô integral
4. model and identify Stochastic differential equations
5. solve linear Stochastic differential equations

Prerequisites: All UG Calculus courses (differential and integral), Measure theory and Lebesgue integration, Propbability, ODE and PDE.

Unit-I: Topics from Probabilty theory (15 hours)

Review of Discrete and Continuous probability model, expectations and Lebesgue integral, Transforms and convergence, independence and covariance, Normal distribution, conditional expectation, Stochastic process in continuous time

Reference: Chapter 2, Introduction To Stochastic Calculus With Applications (3rd Edition) By Fima C Klebaner

Unit-II: Brownian motion (15 hours)
Brownian motion and its properties, hitting times and exit times, maximum and minimum of Brownian motion, distribution of hitting times, reflection principle and joint distribution

Reference: Chapter 3, Introduction To Stochastic Calculus With Applications (3rd Edition) By Fima C Klebaner

Unit-III: Itô Calculus (15 hours)
Random walk, stochastic integral in discrete time, Poisson process, definition of Itô integral, Itô integral process, Itô integral and Gaussian processes, Itô 's formula for Brownian motion

Reference: Chapter 4, Introduction To Stochastic Calculus With Applications (3rd Edition) By Fima C Klebaner

Unit-IV: Stochastic differential equations and applications (15 hours)
Itô processes and stochastic differentials, Itô 's formula for Itô processes, SDEs, Solution to linear SDEs, Existence and uniqueness of strong solutions(Only statement), markov property of solution, weak solutions to SDEs, contruction of weak solution, backward and forward equations, applications

Reference: Chapter 5, Introduction To Stochastic Calculus With Applications (3rd Edition) By Fima C Klebaner

List of Practicals Problems on following topics

1. Convergence in distribution, convergence of expectation
2. Properties of conditional expectation
3. Stopping times, Fubini theorem
4. Properties of Brownian motion
5. Examples of stopping times and random times
6. Application of distribution of maximum and minimum of a Brownian motion
7. Itô integral and its properties
8. Itô integral process, Gaussian process
9. Itô's formula, integration by parts
10. Stochastic differential equations

Recommended Text Books:

1. Michael Steele, Stochastic Calculus and Financial Applications, Applications of Mathematics Springer series in Stochastic modelling and applied probability, vol. 45.
2. Steven E. Shreve, Stochastic Calculus for Finance II Continuous time model, Springer Finance textbook.

3. Daniel J. Stroock, Elements of Stochastic Calculus and Analysis, CRM short courses, Springer.
4. Lawrence C.Evans, An Introduction to Stochastic Differential Equations, AMS.

513016456519: Machine Learning

Course Objectives:

1. To illustrate Regression methods in R.
2. To define Regression diagnostic, unusual observations and corrective measures
3. To implement ANOVA type models.
4. To execute PCA and SVM.

Course Outcomes:

1. Students will recognize linear regression.
2. Students will be able to outline the classification problem.
3. Students will design the fitting and interpretation of ANOVA type models.
4. Students will be able to make predictions with basic types of decision trees.

Prerequisites: Good knowledge of R Programming is necessary for this course.

Unit-I: Regression (15 hours)
Simple Linear Regression, Multiple Linear Regression, Overview of Classification Problem, Logistic Regression, polynomial regression, Regression Diagnostic. Unusual observations (Outliers, High-leverage points, Influential observations). Corrective measures (Deleting observations, Transforming variables, Adding or deleting variables)

Unit-II: Probability and Statistics in R (15 hours)
Linear Discriminant Analysis of $p = 1$ and $p > 1$, Cross Validation, Fitting and interpreting ANOVA type models

Unit-III: Exploratory Data Analysis(EDA) (15 hours)
Power Analysis, T-tests, Creating Power Analysis Plots; Resampling Methods, Bootstrap. Principal components and factor analysis in R. Linear Model Selection and Regularization, Ridge Regression, LASO and PCR.

Unit-IV: Tree Based Methods (15 hours)
Basic Decision trees like Regression Trees, Classification Trees, Bagging, Random Forest, Boosting, Support Vector Machines,

List of Practicals

1. Simple Linear Regression, Multiple Linear Regression,polynomial regression.
2. polynomial regression, Regression Diagnostic.
3. Cross validation.
4. Fitting and interpreting ANOVA type models.
5. Fitting and interpreting ANOVA type models.
6. Creating Power Analysis Plots.
7. Resampling Methods.
8. Resampling Methods, Bootstrap.
9. Principal components and factor analysis.
10. Linear Model Selection.
11. Linear Model Selection and Regularization.
12. Ridge Regression.
13. Regression Trees, Classification Trees.
14. Random Forest.
15. Support Vector Machines.

Recommended Text Books:

1. Gareth James, Witten, Hastie and Tibshirani; *An Introduction to Statistical Learning*; Springer.
2. Nina Zumel and John Mount; *Practical Data Science with R*; Manning publication.
3. Scott Burger; *Introduction to Machine Learning with R*; O'Reilly publication.
4. Robert I. Kabacoff; *R in Action, Data Analysis and Graphics with R and Tidyverse*; Manning publication.

513016456520: Computational Algebra

Course Objectives:

1. To summarize the basic notions of representation theory.
2. To describe the notion of Groebner basis.
3. To perform calculations using free mathematical software.
4. To demonstrate the algorithmic approach towards proof.

Course Outcome:

1. Students will develop the skill of using computers.
2. Previously learnt concepts will be implemented by exploring topics using the mathematical softwares.
3. Students will develop necessary theoretical background before seeing their computer implementation.
4. Students will compute with polynomial rings in many variables and possibly work on research problems in that area in future.

Prerequisites: Students should have done group theory and module theory courses.

Unit-I: Representation Theory (15 hours)
Linear representations of a finite group on a finite dimensional vector space over $\mathbb{C}$. If ρ is a representation of a finite group G on a complex vector space $\mathbb{C}$, then there exists a G-invariant positive definite Hermitian form on V. Complete reducibility (Maschke's theorem). The space of class functions, characters and orthogonality conditions. For a finite group G, there are as many irreducible representations (upto isomorphism) as the number of conjugacy classes in G. Two representations having the same character are isomorphic. Regular representation. Schur's lemma and proof of orthogonality relations. Every irreducible representation over $\mathbb{C}$ of a finite abelian group is one dimensional. Character tables with emphasis on examples of small order.

Unit-II: Group Theory software (15 hours)
Introduction to SAGE Math and GAP softwares. Permutation groups, examples, groups with generators, center of a group, derived series examples, character tables, matrices over finite fields.

Unit-III: Ideals, Varieties and Algorithms (15 hours)
Polynomials in one variable, Affine spaces, Parameterizations of affine spaces, Polynomial rings in more variables, Monomial orderings, Division Algorithm, Dickson's lemma, Hilbert basis theorem, Basics of invariant theory, Groebner basis, Buchberger algorithm and applications.

Unit-IV: Commutative Algebra Software (15 hours)
Introduction to Singular and Macaulay, Polynomials in more than two variables over fields, quotient rings, localizations and Groebner bases.

List of Practicals

1. Examples of representations and invariant subspaces and vectors.
2. Characters of representations.
3. Orthogonality relations and computation of conjugacy classes in known groups.
4. Examples of groups and commands necessary for SAGE MATH and GAP.
5. Examples of center, character tables and commands necessary for SAGE MATH and GAP.
6. Examples of Dickson's lemma and division algorithm in polynomial ring in more variables.
7. Groebner basis and computations in examples.
8. Examples of polynomials in more variables and related commands in software.
9. Examples of computation of Groebner bases in software.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. David A. Cox, John Little and Donald O'Shea, Ideals, Varieties and Algorithms, Springer, 2015.
3. S. Sternberg, Group theory and physics, Cambridge University Press, 1994.
4. Bernd Strumfels, Algorithms in Invariant theory, Springer, 2008.
5. Gordon James and Martin Liebeck, Representations and characters of groups, Second edition, Cambridge.

513016456521: Design Theory

Course Objectives:

1. To develop knowledge of balanced incomplete block designs, which has played an important role in design theory.
2. To relate projective geometry and certain designs.
3. To classify difference families and their constructions.
4. To give examples of Hadamard matrices and model methods for their construction,

Course Outcomes:

1. Students will recognize basic designs and their examples.
2. They will be able to classify affine and projective planes from a design theory point of view.
3. The students will demonstrate construction of difference families.
4. The students will formulate properties of Hadamard matrices.

Unit I. Introduction to Balanced Incomplete Block Designs (15 hours) What Is Design Theory? Basic Definitions and Properties, Incidence Matrices, Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fisher's Inequality.

Unit II. Symmetric BIBDs (15 hours) An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.

Unit III. Difference Sets and Automorphisms (15 hours) Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.

Unit IV. Hadamard Matrices and Designs (15 hours) Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Ma- trices and Hadamard Matrices, A Product Construction, Williamson's Method, Existence Results for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Matrices, Bent Functions.

List of Practicals

1. Examples of designs and other properties.
2. Fisher's inequality and construction of specific designs.
3. Intersection property and examples.
4. Introduction to affine and projective planes and their properties.
5. Examples of difference sets and relation to automorphisms.
6. Multiplier theorem and difference families.
7. Hadamard matrices and their properties.
8. Illustration of different methods of constructions of Hadamard matrices.

Recommended Text Books:

1. D. R. Stinson, Combinatorial Designs: Constructions and Analysis, Springer,2004.
2. W.D. Wallis, Introduction to Combinatorial Designs, (2nd Ed), Chapman & Hall.
3. D. R. Hughes and F. C. Piper, Design Theory, Cambridge University Press, Cambridge, 1985.
4. T. Beth, D. Jungnickel and H. Lenz, Design Theory, Volume 1 (Second Edition), Cambridge University Press, Cambridge, 1999.

Scheme of evaluation R8435 for M. Sc. Mathematics:

A) 100% internal evaluation scheme for University Department of Mathematics. Both Mid and End semester examinations will be conducted by the Department and answer books will be shown to the students.

B) For affiliated PG centers end semester examination will be conducted by the University.

Syllabus Committee

(Ref: AAMS/ICD/2023-24/550 dated 02 Feb. 2024)

Sr. No.	Name	Department/ Institute	Signature
01	Prof. B. S. Desale (Chairman BOS)	University Department of Mathematics	
02	Prof. Vinayak Kulkarni (Co-ordinator)	University Department of Mathematics	
03	Dr. Anuradha Garge (Member BOS)	University Department of Mathematics	
04	Dr. Shridhar Pawar (Member BOS)	Sant Rawool Maharaj Mahavidyalay, Kudal	
05	Prof. R. P. Deore	University Department of Mathematics	
06	Prof. J. V. Prajapat	University Department of Mathematics	
07	Dr. Madhumita Gangopadhyay	University Department of Mathematics	Gangopadhyay
08	Dr. Deepak Sarwe	University Department of Mathematics	
09	Mr. Kamalakar Survade	University Department of Mathematics	
10	Mr. Shantilal Shendage	University Department of Mathematics	

Sign of the BOS Chairman
Dr. Bhausaheb S Desale
The Chairman,
Board of Studies
in Mathematics

Sign of the
Associate Dean
Dr. Madhav R. Rajwade
Faculty of Science
& Technology

Offg. Sign of the Offg. Dean
Prof. Shivram S. Garje
Faculty of Science & Technology