

- N.B. (1) All questions are compulsory.
 (2) Figures to the right indicate marks for respective sub questions.
 (3) Use of **Non-programmable** calculators is **allowed**.
 (4) Draw **neat labeled diagrams** wherever **necessary**.
 (5) Symbols used have their usual meaning

Q.1) Attempt **any THREE** of the following. (15)

- a) Find the adjoint of the given matrix and hence find Inverse if exist

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

- b) Discuss the consistency of the following systems of equations and solve them whenever possible.

$$x + y + z = 3$$

$$2x - y + 3z = 1$$

$$4x + y + 5z = 2$$

$$3x - 2y + z = 4$$

- c) Find the Eigenvalues of the matrix:

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

- d) Find the Modulus and Amplitude of $z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

- e) Express in the form $a+ib$ of $\frac{2-\sqrt{3}i}{1+i}$

- f) If $\sinh x - \cosh x = 5$ then find $\tanh x$.

Q.2) Attempt **any THREE** of the following. (15)

- a) Solve $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

- b) Solve $\sin 2x \frac{dy}{dx} = y + \tan x$

- c) Solve $y = 2px - p^3y^2$

- d) Solve $p^3 + p - e^y = 0$

- e) Solve $(D^4 - 4D^3 + 6D^2 - 4D + 1) = e^x + \frac{1}{3}$

- f) Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = 0$

Q.3) Attempt **any THREE** of the following. (15)

- Find Laplace transform of $f(t) = \sin^2 t$
- Find the Laplace Transform of $f(t) = t e^{3t} \sin 2t$
- If $L[f(t)] = \frac{8+12s-2s^2}{(s^2+4)^2}$ then find $L[f(2t)]$.
- Find the Inverse Laplace Transform of $F(s) = \frac{1}{s^2+6s+5}$
- Find the Inverse Laplace Transform of $F(s) = \frac{3s+1}{(s-4)(s+3)}$.
- Solve the Differential equation $(D^2 + 5D + 6)y = 1, t > 0$ with initial condition $y=y_0, Dy=y_1$ at $t=0$.

Q.4) Attempt **any THREE** of the following. (15)

- Change the order of integration by showing the region of integration

And Evaluate it $\int_0^a \int_{\frac{x}{a}}^{\frac{\sqrt{x}}{a}} (x^2 + y^2) dx dy$

- Evaluate $\int_0^1 \int_0^x x(x^2 + y^2) dx dy$
- Evaluate $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z} dx dy dz$
- Evaluate over positive octant of sphere, the integral $\iiint \frac{dx dy dz}{(1+x^2+y^2+z^2)^2}$
- Evaluate $\int \int y dx dy$ over the area bounded by $x=0, y = x^2$ and $x+y=2$ in the first quadrant
- Find the volume bounded by the cylinder $y^2 = x, x^2 = y$ and the planes $z=0, x+y+z=2$

Q.5) Attempt **any THREE** of the following. (15)

- Evaluate $\int_0^\infty x^2 e^{-h^2 x^2} dx$
- Evaluate $\int_0^1 \frac{x dx}{\sqrt{\log(\frac{1}{x})}}$
- Show that: $\int_0^\infty \frac{1-e^{-ax}}{x} \cdot e^{-x} dx = \log(a+1)$
- Show that: $F(a) = \int_{\pi/6a}^{\pi/2a} \frac{\sin ax}{x} dx$ is independent of a.
- Show that: $\operatorname{erfc}(-x) + \operatorname{erfc}(x) = 2$
- Evaluate: $\frac{d}{dx} [\operatorname{erf}(ax^n) + \operatorname{erfc}(ax)]$.
