Class / Semester / Subject Code: F.Y.B.Sc.(I.T.)/I/USIT104-Discrete Mathematics

Date: 21.11.2019 Time : (2 ½ Hours) Total Marks: 75

N.B. (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective sub questions.
(3) Use of **Non-programmable** calculators is **allowed.**
(4) Draw **neat labeled diagrams** wherever **necessary.**
(5) Symbols used have their usual meaning

Q.1) Attempt **any THREE** of the following. (15)

a Let A = {c, d, f, g}, B = {f, j}, and C = {d, g}.
Answer each of the following questions. Give reasons for your answers.
i. Is B ⊆ A?
ii. Is C ⊆ A?
iii. Is C ⊆ C?
iv. Is C a proper subset of A?

b Define Cartesian product. Let R denote the set of all real numbers.
Describe R x R. let A={1,2,3} and B={a,b,c} find A × B

c Use De Morgan's laws to write negations for the statements below
i. Hellen is a math major and Hellen's sister is a computer science major.
ii. Sam has an orange belt and Kate has a red belt.
iii. The connector is loose or the machine is unplugged.
iv. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.
v. The train is late or my watch is fast.

d Prove That: for all sets A, B and C

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

e Let p: He is rich q: He is happy
Write each of the following statements in symbolic form using p and q.
i. He is poor
ii. If he is rich, then he is unhappy.
iii. He is neither rich nor happy.
iv. It is necessary to be poor in order to be happy.
v. To be poor is to be unhappy

f Define the following:
i. Argument, Premises
ii. Syllogism
iii. Explain Modus Ponens and Modus Tollens with examples

Q.2) Attempt **any THREE** of the following. (15)

a Consider a statement of the form: ∀x ∈ D, if P(x) then Q(x). Write its contrapositive, converse and inverse. Write a formal and an informal contrapositive, converse, and inverse for the following statement:

'If a real number is greater than 2, then its square is greater than 4.'

b Rewrite the statement "No good cars are cheap" in the form "∀x, if P(x) then ~Q(x)." Indicate whether each of the following arguments is valid or invalid, and justify your answers.

a. No good car is cheap.
A Rimbaud is a good car.
∴A Rimbaud is not cheap.

b. No good car is cheap.
A Simbaru is not cheap.
∴A Simbaru is a good car.

c. No good car is cheap.
A VX Roadster is cheap.
∴A VX Roadster is not good

c Show that If r and S are any two rational numbers, then r+S^2 is rational.

d If n is any integer and d is positive integers if
$q = \left\lfloor \frac{n}{d} \right\rfloor$ and $r = n - \left\lfloor \frac{n}{d} \right\rfloor \cdot d$, then n = dq + r and 0 ≤ r < d

e Prove That: $\sqrt{2}$ is irrational.

f State Euclidian Algorithm Find the gcd of (330,156) by using Euclidian Algorithm.

Q.3) Attempt **any THREE** of the following. (15)

a Prove that 7n− 1 is divisible by 6, for each integer n ≥ 0.

b Determine the sequence whose recurrence relation is
$a_n = 4a_{n-1} + 5a_{n-2}$ with $a_1 = 2$ *and* $a_2 = 6$

c Define Arithmetic and geometric sequence.
Let $a_0, a_1, a_2, \ldots$ be the sequence defined recursively as follows:
For all integers k ≥ 1, $a_k = a_{k-1} + 2$ and $a_0 = 1$ Use iteration to guess an explicit formula for the sequence.

d i. Define G: J5 × J5 → J5 × J5 as follows: For all (a, b) ∈ J5 × J5,
$G(a, b) = ((2a + 1) \bmod 5, (3b - 2) \bmod 5)$
Find: $G(4, 4)$, $G(2, 1)$, $G(3, 2)$, $G(1, 5)$
ii. Let F and G be functions from the set of all real numbers to itself.
Define the product functions F •G: R → R and G• F: R → R as follows:
For all x ∈ R,
$(F \bullet G)(x) = F(x) \bullet G(x)$
$(G \bullet F)(x) = G(x) \bullet F(x)$
Does $F \bullet G = G \bullet F$? Explain.

e Define G: R × R → R × R as follows: G(x, y) = (y−2x) for all (x, y) ∈R × R.
i. Is G one-to-one? Prove or give a counterexample.
ii. Is G onto? Prove or give a counterexample.

f Define F: Z → Z and G: Z → Z by the rules $F(a) = 7a$ and $G(a) = a \ mod \ 5$ for all integers a. Find $(G \circ F)(0)$, $(G \circ F)(1)$, $(G \circ F)(2)$, $(G \circ F)(3)$, *and* $(G \circ F)(4)$.

Q.4) Attempt **any THREE** of the following. (15)

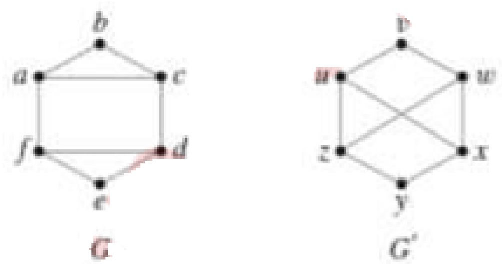
a Let S = {(0, 0), (0, 3), (1, 0), (1, 2), (2, 0), (3, 2)}. Find S^t, the transitive closure of S.

b i. Define inverse relation.
ii. Let A = {3, 4, 5} and B = {4, 5, 6} and let R be the "less than" relation. That is, for all $(x, y) \in A \times B$, $x \ R \ y \Leftrightarrow x < y$. State explicitly which ordered pairs are in R and R^{-1}.

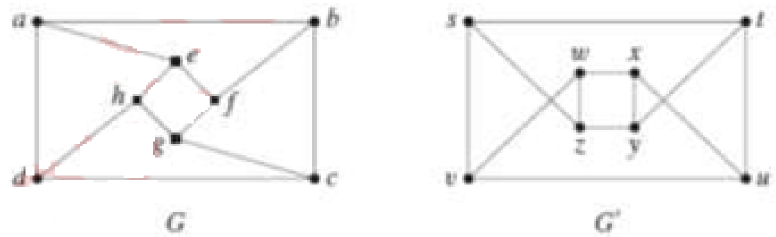
c i. If R and S are reflexive, is R ∩ S reflexive? Why?
ii. If R and S are symmetric, is R ∩ S symmetric? Why?
iii. If R and S are transitive, is R ∩ S transitive? Why?

d Define the following with suitable example :
i) Trail ii) Path iii) Circuit iv) walk v)Tree

e Determine whether G and G' are isomorphic. If they are, give a function g: V(G) → V(G') that defines the isomorphism.
i)



ii)



f For the following either draw the graph as per the specifications or explain why no such graph exists:
i. Graph, circuit-free, nine vertices, six edges
ii. Tree, six vertices, total degree 14
iii. Tree, five vertices, total degree 8

iv. Graph, connected, six vertices, five edges, has a nontrivial circuit
v. Graph, two vertices, one edge, not a tree

Q.5) Attempt **any THREE** of the following. (15)

a A coin is loaded so that the probability of heads is 0.7 and the probability of tails is 0.3. Suppose that the coin is tossed twice and that the results of the tosses are independent.

i. What is the probability of obtaining exactly two heads?
ii. What is the probability of obtaining exactly one head?
iii. What is the probability of obtaining no heads?
iv. What is the probability of obtaining at least one head?

b Team A and team B are play each other repeatedly until one wins two games in a row or a total of three games. One way in which this tournament can be played is for A to win the first game, B to second, and A to win the third and fourth game. Denote this by writing
A-B-A-A

i. How many ways the tournament be played?
ii. Assuming that all the ways of playing the tournament are equally likely, what is probability that five games are needed to determine the tournament winner.

c Suppose that 500000 people pay Rs.5 each to play a lottery game with the following prizes : a grand prize of Rs.1000000 , 10 second prizes of Rs. 1000 each ,1000 third prizes of Rs.500 each, and 10000 fourth prizes prizes of Rs.10 each .What is the expected value of ticket ?

d Prove that for all integers $n \geq 2$,

$$P(n + 1, 2) - P(n, 2) = 2P(n, 1).$$

e An urn contains four balls numbered 2, 2, 5, and 6. If a person selects a set of two balls at random, what is the expected value of the sum of the numbers on the balls?

f A drug-screening test is used in a large population of people of whom 4% actually use drugs. Suppose that the false positive rate is 3% and the false negative rate is 2%. Thus a person who uses drugs tests positive for them 98% of the time, and a person who does not use drugs tests negative for them 97% of the time.

i. What is the probability that a randomly chosen person who tests positive for drugs actually uses drugs?

ii. What is the probability that a randomly chosen person who tests negative for drugs does not use drugs?

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