

Date: 09.10.2019

Time : (2 ½ Hours)

Total Marks: 75

- N.B. (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective sub questions.

Q.1) Attempt All (Each of 5Marks) (15)

(a) Choose correct alternative in each of the following. (5)

(i) A graph with no parallel edges and no loops is called a...graph.

- (a) simple (b) multiple
(c) pseudo (d) none of the above

(ii) The product of two consecutive natural numbers is always divisible by...

- (a) 3 (b) 2
(c) 6 (d) 10

(iii) The Chromatic number of a complete graph on n vertices is...

- (a) n! (c) n
(b) n+1 (d) n-1

(iv) A vertex with degree zero is called as

- (a) Pendent (c) incident
(b) isolated (d) None of the above

(v) Pascal triangle is used to find the coefficient of expansion.

- (a) Multinomial (c) Binomial
(b) Exponential (d) All of the above.

(b) Fill in the blanks (5)

(1,pseudo,degree,combination,zero)

- (i) ... is the selection of r objects from n objects.
(ii) A graph with parallel edges and loop is called as....
(iii) Sink 'T' in network has... out degree.
(iv) The number of edges incident on a vertex is called as...of a vertex.
(v) $n_{c_0} = \dots$

(c) Answer the following in one line (5)

- (i) Clique
(ii) Tree
(iii) Augmenting path
(iv) Binomial theorem
(v) Planar graph

Q.2) Attempt **any THREE** of the following. (15)

(a) Determine the coefficient of $x^2y^2z^2$ in the expansion of $(x + y + z)^6$

(b) By using mathematical induction show that for all positive integers

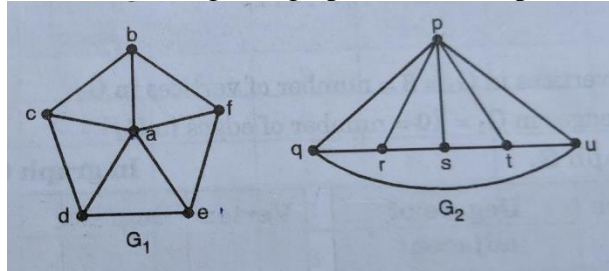
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- (c) How combinatorics and graph theory related to each other? Give an example.
(d) A farmer buys 3 cows, 2 goats and 4 hens from a man who has 4 cows, 3 goats and 8 hens. How many choices dose the farmer have?
(e) If $a_0 = 1, a_1 = 1, a_2 = 1, a_n = a_{n-1} + a_{n-2} + a_{n-3}, \forall n \geq 3$. By using mathematical induction prove that, $a_n \leq 2^{n-1}$.

- (f) How many string of three decimal digits with repetition allowed?
 (A) That begins with an odd number.
 (B) Have exactly two digit that are 4's.

Q.3) Attempt **any THREE** of the following. (15)

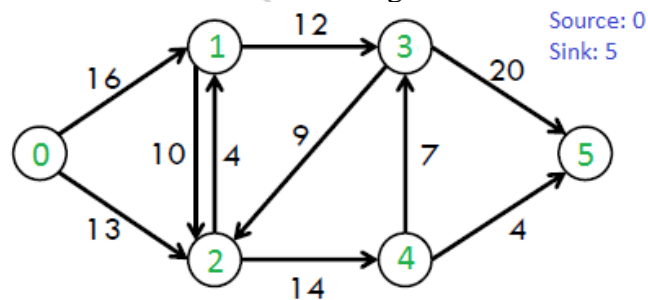
- (a) Check whether the following two graphs are isomorphic or not.



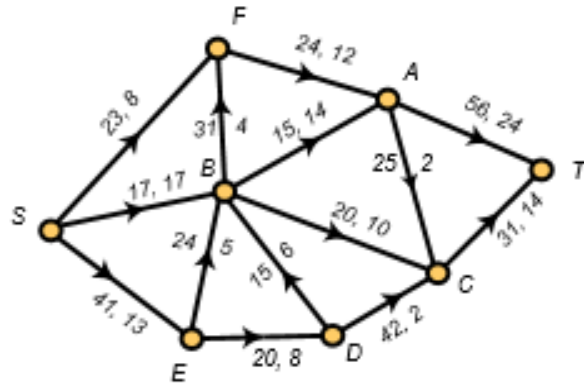
- (b) Draw a tree whose prufer(T)=6643143.
 (c) What is planar graph? Prove that K_4 & $K_{2,2}$ are planar.
 (d) State and prove Euler's formula.
 (e) Determine the number of regions defined by a connected planar graph with 6 vertices and 10 edges. Draw a simple and a non-simple planar graph.
 (f) Define Eulerian and Hamiltonian graphs. Give an example of a graph which is Hamiltonian but not Eulerian.

Q.4) Attempt **any THREE** of the following. (15)

- (a) Find the maximum value of the following network.



- (b) What is integer solution of a linear programming problem?
 (c) Explain matching in bipartite graph.
 (d) Suppose we are coloring the vertices of the square using black and white. Draw all the possible pattern of coloring also find the different transformations for fixed coloring.
 (e) Find flow of the following network and find any two cuts in the network also find there capacities.

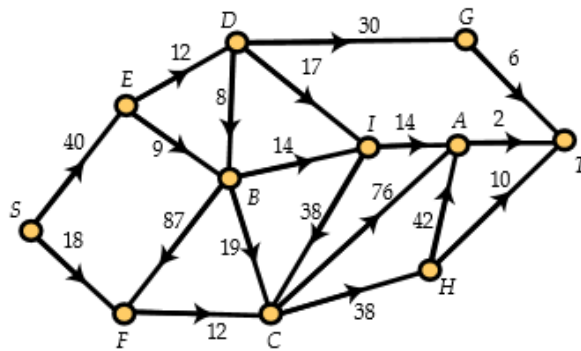


- (f) Write permutations shown below in cycle notation of π_1 and π_2 also compute $\pi_1\pi_2$ (product of two permutations)

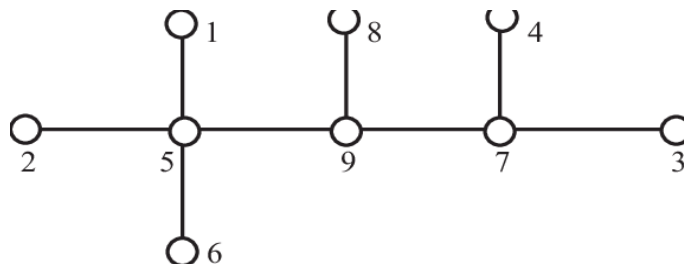
$$\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 5 & 6 & 2 & 1 \end{pmatrix}, \pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 3 & 4 & 2 \end{pmatrix}$$

Q.5) Attempt **any THREE** of the following. (15)

- What is graph colouring? What is chromatic number of a graph? Explain with a suitable example.
- Using mathematical induction prove that, $2^n > n^2$ for $n \geq 5$ $n \in \mathbb{N}$
- Find the maximum flow of the following network by using Ford-Fulkerson's algorithm.



- (d) Determine Prufer code for the following tree.



- (e) Prove that in any finite undirected graph the number of vertices of odd degree is always even.