CIRCULAR:-

Attention of the Head, University Department of Mathematics and the Principals of the Affiliated Colleges in Arts & Science faculties is invited to this office circular No.UG/28 of 2006, dated 27th January, 2006 relating to syllabus of M.A./M.Sc. degree courses are hereby informed that the recommendations made by the Board of Studies in Mathematics under the Faculty of Science and Technology at its meeting held on 17th June, 2017 have been accepted by the Academic Council at its meeting held on 30th July, 2017 vide item No. 4.1 and that in accordance therewith, the revised syllabus as per the (CBCS) for M.A./M.Sc. in Mathematics (Sem. III & IV), has been brought into force with effect from the academic year 2018-19, accordingly. (The same is available on the University’s web site: www.mu.ac.in).

MUMBAI – 400 032
14th December,, 2017

To,

The Head, University Department of Mathematics and the Principals of affiliated Colleges in Arts and Science.

A.C/4.1/30/07/2017

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No. UG/318-A of 2017-18
MUMBAI-400 032
14th December,, 2017

Copy forwarded with compliments for information to:-

1) The Co-Ordinator, Faculty of Arts & Humanities and Science & Technology.
2) The Chairperson, Board of Studies in Mathematics,
3) The Director, Board of Examinations and Evaluation,
4) The Director, Board of Students Development,
5) The Professor-cum-Director, Institute of Distance and Open Learning (IDOL),
6) The Co-Ordinator, University Computerization Centre.

(Dr.Dinesh Kamble)
I/c REGISTRAR
Syllabus
for
M.A./M.Sc. Semester III & IV (CBCS)
Program: M.A/M.Sc.
Course: Mathematics
with effect from the academic year 2018-2019
## M.A./M.Sc. Semester III and IV
Choice Based Credit System (CBCS)

### Semester III

#### Algebra III

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT301, PAMT301</td>
<td>Unit I</td>
<td>Groups</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Unit II</td>
<td>Representation of finite groups</td>
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<td>Unit III</td>
<td>Modules</td>
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<td></td>
<td>Unit IV</td>
<td>Modules over PID</td>
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#### Functional Analysis

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<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT302, PAMT302</td>
<td>Unit I</td>
<td>Baire spaces, Hilbert spaces ( \mathbb{R}^n )</td>
<td>6</td>
<td>4</td>
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<tr>
<td></td>
<td>Unit II</td>
<td>Normed linear spaces</td>
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<td></td>
<td>Unit III</td>
<td>Bounded linear maps</td>
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<td></td>
<td>Unit IV</td>
<td>Basic theorems</td>
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#### Differential Geometry

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT303, PAMT303</td>
<td>Unit I</td>
<td>Isometries of ( \mathbb{R}^n )</td>
<td>6</td>
<td>4</td>
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<tr>
<td></td>
<td>Unit II</td>
<td>Curves</td>
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<tr>
<td></td>
<td>Unit III</td>
<td>Regular surfaces</td>
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<td></td>
<td>Unit IV</td>
<td>Curvature</td>
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#### Elective Courses

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT304, PAMT304</td>
<td>Elective Course I</td>
<td></td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PSMT305, PAMT305</td>
<td>Elective Course II</td>
<td></td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

### Note:

1. PSMT301/PAMT301, PSMT302/PAMT302, PSMT303/PAMT303 are compulsory courses for Semester III.
2. PSMT 304/PAMT 304 and PSMT 305/PAMT305 are Elective Courses for Semester III.
3. Elective course Courses I and II will be any TWO of the following list of ten courses:
1. Four lectures per week for each of the courses: PSMT301/PAMT301, PSMT302/PAMT302, PSMT303/PAMT303, PSMT304/PAMT304 and PSMT305/PAMT305. Each lecture is of 60 minutes duration.

2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

3. The lectures of the Skill Course are held on Sundays or other Holidays. This course shall be approximately 100 hours duration. 75% attendance is mandatory for this course.

Teaching Pattern for Semester III
## Semester IV

### Field Theory

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT401,PAMT401</td>
<td>Unit I</td>
<td>Algebraic Extensions</td>
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<td></td>
<td>Unit II</td>
<td>Normal and Separable Extensions</td>
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<td></td>
<td>Unit III</td>
<td>Galois theorems</td>
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<td>Unit IV</td>
<td>Applications</td>
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### Fourier Analysis

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<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
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<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT402,PAMT402</td>
<td>Unit I</td>
<td>Fourier series</td>
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<td></td>
<td>Unit II</td>
<td>Dirichlet's theorem</td>
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<td></td>
<td>Unit III</td>
<td>Fejer’s theorem and applications</td>
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<tr>
<td></td>
<td>Unit IV</td>
<td>Dirichlet theorem in the unit disc</td>
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### Calculus on Manifolds

<table>
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<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT403,PAMT403</td>
<td>Unit I</td>
<td>Multilinear Algebra</td>
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<td></td>
<td>Unit II</td>
<td>Differential Forms</td>
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<td>Unit III</td>
<td>Basics of Submanifolds of $\mathbb{R}^n$</td>
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<td>Unit IV</td>
<td>Stokes’ Theorem and applications</td>
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<td>5</td>
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### Optional Course

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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</thead>
<tbody>
<tr>
<td>PSMT404,PAMT404</td>
<td>OC 1: Optional Course I</td>
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<tr>
<td></td>
<td>OC 2: Optional Course II</td>
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<td>4</td>
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### Project Course

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<tr>
<th>Course Code</th>
<th>Unit</th>
<th>Topics</th>
<th>Credits</th>
<th>L/W</th>
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<tbody>
<tr>
<td>PSMT405,PAMT405</td>
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<td>Project Course</td>
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Note:

1. PSMT401/PAMT401, PSMT402/PAMT402, PSMT403/PAMT403 are compulsory courses for Semester IV.

2. PSMT 404/PAMT 404 is an Optional Course for Semester IV. This is a Choice Based Course.
3. PSMT 405/PAMT 405 is a project based Course for Semester IV. The projects for this course are to be guided by the Faculty members of the Department of Mathematics of the concerned college. Each project shall have maximum of 08 (eight) students. The workload for each project is 1L/W.

Teaching Pattern for Semester IV

1. Four lectures per week for each of courses: PSMT401/PAMT401, PSMT402/PAMT402, PSMT403/PAMT403 & PSMT404/PAMT404. Each lecture is of 60 minutes duration. In addition, there shall be tutorials, seminars as necessary for each course.

SEMESTER III

All Results have to be done with proof unless otherwise stated.

PSMT301, PAMT301  Algebra III

Unit I. Groups  (15 Lectures)
Simple groups, $A_5$ is simple.
Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Zassenhaus Lemma, Jordan-Holder theorem,
   Direct and Semi-direct products, Examples such as
   (i) The group of affine transformations $x \mapsto ax + b$ as semi-direct product of the group of linear transformations acting on the group of translations.
   (ii) Dihedral group $D_{2n}$ as semi-direct product of $\mathbb{Z}_2$ and $\mathbb{Z}_n$.
Classification of groups of order 12. (Ref: M. Artin, Algebra)

Unit II. Representation of finite groups (15 Lectures)
Linear representations of a finite group over a finite dimensional vector space over $\mathbb{C}$. If $\rho$ is a representation of a finite group $G$ on a complex vector space $V$, then there exits a $G$-invariant positive definite Hermitian inner product on $V$. Complete reducibility (Maschke’s theorem).
   The space of class functions, Characters and Orthogonality relations. For a finite group $G$, there are finitely many isomorphism classes of irreducible representations, the same number as the number of conjugacy classes in $G$. Two representations having same character are isomorphic. Regular representation. Schur’s lemma and proof of the Orthogonality relations. Every irreducible representation over $\mathbb{C}$ of a finite Abelian group is one dimensional.
   Character tables with emphasis on examples of groups of small order.
Reference for Unit II:

2. S. Sternberg, Group theory and Physics, Cambridge University Press.
Unit III. Modules (15 Lectures)


Free modules, free module of rank $n$. For a commutative ring $R$, $R^n$ is isomorphic to $R^m$ if and only if $n = m$. Matrix representations of homomorphisms between free modules of finite ranks. (Ref: N. Jacobson, *Basic Algebra*, Volume 1.)

Dimension of a free module over a P.I.D. (ref: S. Lang, *Algebra*).

Unit IV. Modules over PID (15 Lectures)

Finitely generated modules over a PID: If $N$ is a submodule of free module $M$ (over a P.I.D.) of finite rank $n$, then $N$ is free of rank $m \leq n$. Any submodule of a finitely generated module over a P.I.D. is finitely generated. (ref: S. Lang, *Algebra*)


Recommended Text Books:

2. S. Lang, *Algebra*, Springer Verlag, 2004

PSMT302, PAMT302 Functional Analysis

Unit I Baire spaces, Hilbert spaces (15 Lectures)
Baire spaces. Open subspace of a Baire space is a Baire space. Complete metric spaces are Baire spaces and application to a sequence of continuous real valued functions converging point-wise to a limit function on a complete metric space. (ref: *Topology* by J.R. Munkres)

Hilbert spaces, examples of Hilbert spaces such as $l^2, L^2(-\pi, -\pi), L^2(\mathbb{R}^n)$ (with no proofs). Bessel’s inequality. Equivalence of complete orthonormal set and maximal orthonormal basis. Orthogonal decomposition. Existence of a maximal orthonormal basis. Parseval’s identity. Riesz Representation theorem for Hilbert spaces. (ref: *Introduction to Topology and Modern Analysis* by G. F. Simmons)

Unit II. Normed Linear Spaces (15 Lectures)
Normed Linear spaces. Banach spaces. Quotient space of a normed linear space. $l^p$ ($1 \leq p \leq \infty$) spaces are Banach spaces.

$L^p(\mu)$ ($1 \leq p \leq \infty$) spaces: Holder’s inequality, Minkowski’s inequality, $L^p(\mu)$ ($1 \leq p \leq \infty$) are Banach spaces (ref: ROYDEN, Real Analysis).

Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and application to infinite dimensional normed linear spaces (ref: E. KERYSZIG, Introductory Functional Analysis with Applications).

**Unit III. Bounded Linear Transformations** (15 Lectures)
Bounded linear transformations, Equivalent characterizations. The space $\mathcal{B}(X,Y)$. Completeness of $\mathcal{B}(X,Y)$ when $Y$ is complete. Hahn-Banach theorem, dual space of a normed linear space, applications of Hahn-Banach theorem.
Reference for unit II: E. KERYSZIG, Introductory Functional Analysis with Applications).

**Unit IV. Basic Theorems** (15 Lectures)

Separable spaces, examples of separable spaces such as $l^p(1 \leq p < \infty)$. If the dual space $X'$ of $X$ is separable, then $X$ is separable (ref: B.V. LIMAYE, Functional Analysis).

Dual spaces of $l^p(1 \leq p < \infty)$ (ref: E. KERYSZIG, Introductory Functional Analysis with Applications)

Dual of $L^p(\mu)(1 \leq p < \infty)$ spaces: Riesz-Representation theorem for $L^p(\mu)(1 \leq p < \infty)$ spaces (ref: ROYDEN, Real Analysis).

**Recommended Text Books:**
3. ROYDEN, Real Analysis, Macmillian.

**PSMT303, PAMT303 Differential Geometry**

**Unit I. Isometries of $\mathbb{R}^n$** (15 Lectures)
Orthogonal transformations of $\mathbb{R}^n$ and Orthogonal matrices. Any isometry of $\mathbb{R}^n$ fixing the origin is an orthogonal transformation. Any isometry of $\mathbb{R}^n$ is the composition of an orthogonal transformation and a translation. Orientation preserving isometries of $\mathbb{R}^n$.

Reflection map about a hyperplane $W$ of $\mathbb{R}^n$ through the origin: Let $W$ be a vector subspace
of $\mathbb{R}^n$ of dimension $n - 1$. Let $n$ be any unit vector in $\mathbb{R}^n$ orthogonal to $W$. Define $T(v) = v - 2\langle v, n \rangle n$, ($v \in \mathbb{R}^n$). Then $T$ is an orthogonal transformation of $\mathbb{R}^n$, and $T$ is independent of the choice of $n$. Any isometry of $\mathbb{R}^n$ is the composition of at most $n + 1$ many reflections.

Isometries of the plane: Rotation map of $\mathbb{R}^2$ about any point $p$ of $\mathbb{R}^2$, reflection map of $\mathbb{R}^2$ about any line $l$ of $\mathbb{R}^2$. Glide reflection of $\mathbb{R}^2$ (obtained by reflecting about a line $l$ and then translating by a non-zero vector $v$ parallel to $l$). Any isometry of $\mathbb{R}^2$ is a rotation, a reflection, a glide reflection, or the identity.

References for Unit I:
1. S. Kumaresan, *A Course in Riemannian geometry*.
2. M. Artin, *Algebra*, PHI.

Unit II. Curves (15 Lectures)
Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Signed curvature for plane curves, Curvature and torsion of curves in $\mathbb{R}^3$ and their invariance under orientation preserving isometries of $\mathbb{R}^3$. Serret-Frenet equations. Fundamental theorem for space curves in $\mathbb{R}^3$.

Unit III. Regular Surfaces (15 Lectures)
Regular surfaces in $\mathbb{R}^3$, Examples. Surfaces as level sets, Surfaces as graphs, Surfaces of revolution. Tangent space to a surface at a point, Equivalent definitions. Smooth functions on a surface, Differential of a smooth function defined on a surface. Orientable surfaces. Mobius band is not orientable.

Unit IV. Curvature (15 Lectures)
The first fundamental form. The Gauss map, the shape operator of a surface at a point, self-adjointness of the shape operator, the second fundamental form, Principle curvatures and directions, Euler’s formula, Meusnier’s Theorem, Normal curvature. Gaussian curvature and mean curvature, Computation of Gaussian curvature, Isometries of surfaces, Covariant differentiation, Gauss’s Theorema Egregium (statement only), Geodesics.

Recommended Text Books:
5. S. Kumaresan, *A Course in Riemannian geometry*. 
The Elective Courses I and II will be any TWO of the following list of ten courses:

1. Algebraic Topology

Unit I. Fundamental Group (15 Lectures)

Unit II. Fundamental group, Applications (15 Lectures)

Unit III. Covering Spaces (15 Lectures)

Unit IV. Simplicial Homology (15 lectures)
$\Delta$-Complexes, Simplicial Homology, computation of simplicial Homology groups for $S^2, T^2$.

Recommended Text Books:

2. Advanced Complex Analysis

Unit I. Monodromy (15 Lectures)
Holomorphic functions of one variable. Germs of holomorphic functions. Analytic continuation along a path. Examples including $z^{1/n}$ and $\log(z)$, Homotopy between paths. The monodromy theorem.

Unit II. Riemann Mapping Theorem (15 Lectures)
Uniform convergence, Ascoli’s theorem, Riemann mapping theorem.

Unit III. Elliptic Functions (15 Lectures)
Lattices in $\mathbb{C}$. Elliptic functions (doubly periodic meromorphic functions) with respect to a lattice.
Sum of residues in a fundamental parallelogram is zero and the sum of zeros and poles (counting multiplicities) in a fundamental parallelogram is zero, Weierstrass $P$-function, Relation between $P$ and $P'$, Theorem that $P$ and $P'$ generate the field of elliptic functions.

Unit IV. Zeta Function (15 Lectures)
Gamma and Riemann Zeta functions, Analytic continuation, Functional equation for the Zeta function.

Recommended Text Books:
1. S. Lang, *Complex Analysis*, Springer.
2. John Conway, *Functions of one complex variable*, Narosa India.

3. Commutative Algebra

Unit I. Basics of rings and modules (15 Lectures)
Basic operations with commutative rings and modules, Polynomial and power series rings, Prime and maximal ideals, Extension and contractions, Nil and Jacobson radicals, Chain conditions, Hilbert basis theorem, Localization, Local rings, Nakayama’s lemma, Tensor products.

Unit II. Primary decomposition (15 Lectures)
Associated primes, Primary decomposition.

Unit III. Integral Extensions (15 Lectures)
Integral extensions, Going up and going down theorems, The ring of integers in a quadratic extension of rationals, Noether normalization, Hilbert’s nullstellensatz.

Unit IV. Dedekind Domains (15 Lectures)
Artinian rings, Discrete valuation rings, Alternative characterizations of discrete valuation rings, Dedekind domains, Fractional ideals, Factorization of ideals in a Dedekind domain, Examples.

Recommended Text Books:
1. S. Lang, *Complex Analysis*, Springer.

4. Algebraic Number Theory

Unit I. Number Fields (15 Lectures)
Field extensions, Number fields, Algebraic numbers, Integral extensions, Ring of integers in a
number field, Fractional ideals, Prime factorization of ideals, Norm of an ideal, Ideal classes, The class group, The group of units.

Unit II. Quadratic Reciprocity (15 Lectures)
The Legendre symbol, Jacobi symbols, The laws of quadratic reciprocity.

Unit III. Quadratic Fields: Factorization (15 Lectures)
Quadratic fields, Real and imaginary quadratic fields, Ring of integers in a quadratic field, The group of units, Ideal Factorization in a quadratic field, Examples: The ring of Gaussian integers, The ring \( \mathbb{Z}[\sqrt{5}] \), Factorization of rational primes in quadratic fields.

Unit IV. Imaginary Quadratic Fields: The Class Group (15 Lectures)
The ideal class group of a quadratic field, Class groups of imaginary quadratic fields, The Minkowski lemma, The finiteness of the class group, Computation of class groups, Application to Diophantine equations.

Recommended Text Books:


5. Partial Differential Equations

Unit I. Classification of second order Linear partial differential equations (15 Lectures)

The classification of second order linear partial differential equations.

Unit II. Laplace operator (15 Lectures)
Symmetry properties of the Laplacian, basic properties of the Harmonic functions, the Fundamental solution, the Dirichlet and Neumann boundary value problems, Green's function. Applications to the Dirichlet problem in a ball in \( \mathbb{R}^n \) and in a half space of \( \mathbb{R}^n \). Maximum Principle
for bounded domains in \( \mathbb{R}^n \) and uniqueness theorem for the Dirichlet boundary value problem.

**Unit III. Heat operator** (15 Lectures)

The properties of the Gaussian kernel, solution of initial value problem \( u_t - \Delta u = 0 \) for \( x \in \mathbb{R}^n \) & \( t > 0 \) and \( u(x,0) = f(x) (x \in \mathbb{R}^n) \). Maximum principle for the heat equation and applications.

**Unit IV. Wave operator** (15 Lectures)

Wave operator in dimensions 1, 2 & 3; Cauchy problem for the wave equation. DAlemberts solution, Poisson formula of spherical means, Hadamards method of descent, Inhomogeneous Wave equation.

**Recommended Text Books:**


**6. Numerical Analysis**

**Unit I. Basics of Numerical Analysis** (15 Lectures)

Representation of numbers: Binary system, Hexadecimal system, octal system. Ones complement, twos complement in binary application for subtraction. Russian Peasants method for multiplication and its application in binary system for multiplication.

Errors in numerical computation of numbers: Floating point representation of numbers, rounding off errors and Mantissa & exponent, Truncation errors, Inherent errors.


**Unit II. Numerical linear algebra** (15 Lectures)

Gauss elimination to obtain LU factorization of matrices and partial pivoting in matrices.

Gauss-Jacobi and Gauss-Siedel methods for solving system of linear equations with derivation of convergence.

Greschgorin theorem and Brower’s theorem for bounds of eigenvalues of matrices.

**Unit III. Roots of equations** (15 Lectures)

Only the methods listed below are expected.

Bisection method with proof of convergence and derivation of and rate of convergence, Regula
Falsi and secant methods, Newton-Raphson method. Rates of convergence, sufficient condition for convergence of iteration scheme and application to Newton-Raphson method.

Ramanujan’s method, Muller’s method for detection of complex roots, Berge-Vieta and Bairstwo methods for roots of polynomials.

Unit IV. Numerical Integration (15 Lectures)
Lagrange’s interpolation formula, uniqueness of interpolation, general error in interpolation (No other interpolation formulae expected).

Trapezoidal and Simpson’s $\frac{1}{3}$-rule in composite forms, Gauss Legendre numerical integration, Gauss-Chebyshev numerical integration, Gauss-Hermite numerical integration, Gauss-Laguerre numerical integration with the derivation of all methods using the method of undetermined coefficients.

Estimation of error in numerical integration by using error constant method as in [1]. Only the following seven methods are expected using C and C++ Programs: 1) Bisection method 2) Newton-Raphson method 3) Gauss-Jacobi method 4) Gauss-Siedel method 5) Trapezoidal rule 6) Simpson’s rule 7) Muller’s method.

Recommended Text books:
2. S.S. Sastry, Numerical Methods, Prentice-Hall India.
3. V. Rajaram, Computer Oriented Numerical Methods, Prentice Hall India.

7. Graph Theory

Unit I. Connectivity (15 Lectures)
Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra’s algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa' \leq \delta$, Blocks, Block-cut point theorem, Construction of reliable communication network, Menger’s theorem.

Unit II. Trees (15 Lectures)

Unit III. Eulerian and Hamiltonian Graphs (15 Lectures)
Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury’s algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac’s theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

Unit IV. Matching and Ramsey Theory (15 Lectures)
Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Halls theorem, Konig’s theorem, Tutte’s theorem, Personal assignment problem, Independent sets and covering- \( \alpha + \beta = p \), Gallai’s theorem, Ramsey theorem-Existence of \( r(k, l) \). Upper bounds of \( r(k, l) \), Lower bound for \( mr(k, l) \geq 2^{m/2} \) where \( m = \min\{k, l\} \), Generalize Ramsey numbers- \( r(k_1, k_2, \ldots, k_n) \), Graph Ramsey theorem, Evaluation of \( r(G, H) \) when for simple graphs \( G = P_3, H = C_4 \).

Recommended Text Books:
1. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier.

8. Design Theory

Unit I. Introduction to Balanced Incomplete Block Designs (15 Lectures)

Unit II. Symmetric BIBDs (15 Lectures)
An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.

Unit III. Difference Sets and Automorphisms (15 Lectures)

Unit IV. Hadamard Matrices and Designs (15 Lectures)
Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Matrices and Hadamard Matrices, A Product Construction, Williamson’s Method, Existence Results.
for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Matrices, Bent Functions.

**Recommended Text Books:**


**9. Coding Theory**

Unit I. Error detection, Correction and Decoding (15 Lectures)
Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor/minimum distance decoding, Distance of a code.

Unit II. Linear codes (15 Lectures)
Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbour decoding for linear codes, Syndrome decoding.

Unit III. Cyclic codes (15 Lectures)
Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

Unit IV. Some special cyclic codes (15 Lectures)
Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

**Recommended Text Books:**

1. San Ling and Chaoing Xing, *Coding Theory- A First Course*.

**10. Integral Transforms**

Unit I. Laplace Transform (15 Lectures)
Definition of Laplace Transform, Laplace transforms of some elementary functions, Properties of Laplace transform, Laplace transform of the derivative of a function, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Inverse Laplace Transform of derivatives, Convolution
Theorem, Heaviside’s expansion theorem, Application of Laplace transform to solutions of ODEs and PDEs.

**Unit II. Fourier Transform** (15 Lectures)
Fourier Integral theorem, Properties of Fourier Transform, Inverse Fourier Transform, Convolution Theorem, Fourier Transform of the derivatives of functions, Parseval’s Identity, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems.

**Unit III. Mellin Transform** (15 Lectures)
Properties and evaluation of Mellin transforms, Convolution theorem for Mellin transform, Complex variable method and applications.

**Unit IV. Z-Transform** (15 Lectures)
Definition of Z-transform, Inversion of the Z-transform, Solutions of difference equations using Z-transform.

**Recommended Text Books:**

**Skill Course**
The Skill Course is any one of the following three courses:

**Skill Course I: Business Statistics**

**Unit I. Data Classification, Tabulation and Presentation** (15 Lectures)
Classification of Data: Requisites of Ideal Classification, Basis of Classification.

Organizing Data Using Data Array: Frequency Distribution, Methods of Data Classification, Bivariate Frequency Distribution, Types of Frequency Distributions.

Tabulation of Data: Objectives of Tabulation, Parts of a Table, Types of Tables, General and Summary Tables, Original and Derived Tables.

Graphical Presentation of Data: Functions of a Graph, Advantages and Limitations of Diagrams
General Rules for Drawing Diagrams.

Types of Diagrams: One-Dimensional Diagrams, Two-Dimensional Diagrams, Three-Dimensional Diagrams, Pictograms or Ideographs, Cartograms or Statistical Maps.

Exploratory Data Analysis: Stem-and-Leaf Displays.

Unit II. Measures of Central Tendency (15 Lectures)

Objectives of Averaging, Requisites of a Measure of Central Tendency, Measures of Central Tendency,

Mathematical Averages: Arithmetic Mean of Ungrouped Data, Arithmetic Mean of Grouped (Or Classified) Data, Some Special Types of Problems and Their Solutions, Advantages and Disadvantages of Arithmetic Mean, Weighted Arithmetic Mean.

Geometric Mean: Combined Geometric Mean, Weighted Geometric Mean, Advantages, Disadvantages and Applications of G.m.

Harmonic Mean: Advantages, Disadvantages and Applications of H.M. Relationship Between A.M., G.M. and H.M.

Averages of Position: Median, Advantages, Disadvantages and Applications of Median.


Relationship Between Mean, Median and Mode, Comparison Between Measures of Central Tendency.

Unit III. Measures of Dispersion (15 Lectures)

Significance of Measuring Dispersion (Variation): Essential Requisites for a Measure of Variation. Classification of Measures of Dispersion.

Distance Measures: Range, Interquartile Range or Deviation.

Average Deviation Measures: Mean Absolute Deviation, Variance and Standard Deviation, Mathematical Properties of Standard Deviation, Chebyshev’s Theorem, Coefficient of Varia-
Unit IV. Skewness, Moments and Kurtosis (15 Lectures)

Measures of Skewness: Relative Measures of Skewness.
   Moments: Moments About Mean, Moments About Arbitrary Point, Moments About Zero or Origin, Relationship Between Central Moments and Moments About Any Arbitrary Point, Moments in Standard Units, Sheppard’s Corrections for Moments.
   Kurtosis: Measures of Kurtosis.

Reference Book:

Skill Course II: Statistical Methods

Unit I. Basic notions of Statistics (15 Lectures)

Measures of central tendencies: Mean, Median, Mode.


Measures of relationship: Covariance, Karl Pearson’s coefficient of Correlation, Rank Correlation.

   Basics of Probability.

Unit II. Sampling and Testing of Hypothesis (15 Lectures)


Unit III. Analysis of Variance (15 Lectures)

Reference for Unit III: Chapter 12 of the book: C. R. Kothari and G. Garg, Research
Methodology Methods and Techniques, New Age International.

Unit IV. Use of package R (15 Lectures)

R as Statistical software and language, methods of Data input, Data accessing, useful built-in functions, Graphics with R, Saving, storing and retrieving work.

Skill Course III: Computer Science

Aim: Mathematics students are well versed in logic. This Skill course aims at giving input of necessary skills of algorithms and data structures and relational database background so that the students are found suitable to be absorbed as trainee software professional in industry.
Prerequisite for this course: Good knowledge of C, C++ or java or python.

Unit I. OOPS Concepts (15 lectures)
Basics of object oriented programming principles, templates, reference operators NEW and delete in C++, the java innovation which avoids use of delete, classes polymorphism friend functions, inheritance, multiple inheritance operator overloading basics only references for Unit I: [3], [4]

Unit II. Basic Algorithms (15 lectures)
Basic algorithms, selection sort, quick sort, heap sort, priority queues, radix sort, merge sort, dynamic programming, app pairs, shortest paths, image compression, topological sorting, single source shortest paths reference, hashing intuitive evaluation of running time.
references:[1], [2]

Unit III. Data Structures (15 lectures)
Stacks queues, linked lists implementation and simple applications, trees implementation and tree traversal (stress on binary trees),

Unit IV. Relational Databases (15 lectures)
concept of relational databases, normal forms BCNF and third normal forms. Armstrongs axioms. Relational algebra and operations in it.
references: [5]

recommended Text Books:

1. T. Aron and others, Data structure using C.
2. S. Sahani, Data structures and applications, TMH.

5. J.D. Ullam, *Principles of Database systems*.

**SEMESTER IV**

**PSMT401, PAMT401  Field Theory**

**Unit I. Algebraic Extensions** (15 lectures)
Revision: Prime subfield of a field, definition of field extension $K/F$, algebraic elements, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element.

Algebraic extensions, Finite extensions, degree of an algebraic element, degree of a field extension. If $\alpha$ is algebraic over the field $F$ and $m_\alpha(x)$ is the minimum polynomial of $\alpha$ over $F$, then $F(\alpha)$ is isomorphic to $F[X]/(m_\alpha(x))$. If $F \subseteq K \subseteq L$ are fields, then $[L : F] = [L : K][K : F]$.

If $K/F$ is a field extension, then the collection of all elements of $K$ which are algebraic over $F$ is a subfield of $K$. If $L/K, K/F$ are algebraic extensions, then so is $L/F$. Composite field $K_1K_2$ of two subfields of a field and examples. (Ref: D.S. Dummit and R.M. Foote, *Abstract Algebra*)

Classical Straight-edge and Compass constructions: definition of Constructible points, lines, circles by Straight-edge and Compass starting with $(0,0)$ and $(1,0)$, definition of constructible real numbers. If $a \in \mathbb{R}$ is constructible, then $a$ is an algebraic number and its degree over $\mathbb{Q}$ is a power of 2. $\cos 20^\circ$ is not a constructible number. The regular 7-gon is not constructible. The regular 17-gon is constructible. The Constructible numbers form a subfield of $\mathbb{R}$. If $a > 0$ is constructible, then so is $\sqrt{a}$. (Ref: M. Artin, *Algebra*, Prentice Hall of India)

Impossibility of the classical Greek problems: 1) Doubling a Cube, 2) Trisecting an Angle, 3) Squaring the Circle is possible. (Ref: D.S. Dummit and R.M. Foote, *Abstract Algebra*)

**Unit II. Normal and Separable Extensions** (15 lectures)
Splitting field for a set of polynomials, normal extension, examples such of splitting fields of $x^p - 2$ ($p$ prime), uniqueness of splitting fields, existence and uniqueness of finite fields. Algebraic closure of a field, existence of algebraic closure.

Separable elements, Separable extensions. In characteristic $0$, all extensions are separable. Frobenius automorphism of a finite field. Every irreducible polynomial over a finite field is separable. Primitive element theorem.

**Unit III. Galois Theory** (15 Lectures)
Galois group $G(K/F)$ of a field extension $K/F$, Galois extensions, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory.

**Unit IV. Applications** (15 Lectures)

Cyclotomic field $\mathbb{Q}(\zeta_n)$ (splitting field of $x^n - 1$ over $\mathbb{Q}$), cyclotomic polynomial, degree of Cyclotomic field $\mathbb{Q}(\zeta_n)$. D.S. Dummit and R.M. Foote, *Abstract Algebra*

Galois group for an irreducible cubic polynomial, Galois group for an irreducible quartic polynomial. (Ref: M. Artin, *Algebra*, Prentice Hall of India)

Solvability by radicals in terms of Galois group and Abel’s theorem on the insolvability of a general quintic. (Ref: D.S. Dummit and R.M. Foote, *Abstract Algebra*)

**Recommended Text Books:**


**Additional Reference Books:**

1. S. Lang, *Algebra*, Springer Verlag, 2004  

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**PSMT402, PAMT402 Fourier Analysis**

**Unit I. Fourier series** (15 Lectures)

The Fourier series of a periodic function, Dirichlet kernel, Bessel’s inequality for a $2\pi$-periodic Riemann integrable function, convergence theorem for the Fourier series of a $2\pi$-periodic and piecewise $C^1$- function, uniqueness theorem (If $f, g$ are $2\pi$-periodic and piecewise smooth function having same Fourier coefficients, then $f = g$).

Relating Fourier coefficients of $f$ and $f'$ where $f$ is continuous $2\pi$-periodic and piecewise $C^1$- function and a convergence theorem: If $f$ is continuous $2\pi$-periodic and piecewise $C^1$-function, then the Fourier series of $f$ converges to $f$ absolutely and uniformly on $\mathbb{R}$.


**Unit II. Dirichlet’s theorem** (15 Lectures)

Review: Lebesgue measure of $\mathbb{R}$, Lebesgue integrable functions, Dominated Convergence theorem, Bounded linear maps (no questions be asked).

Definition of Lebesgue integrable periodic functions (i.e. $L^1$-periodic), Fourier Coefficients of $L^1$-periodic functions, $L^2$-periodic functions. Any $L^2$-periodic function is $L^1$-periodic. Riemann-Lebesgue Lemma (if $f$ is Lebesgue integrable periodic function, then $\lim_{|n| \to \infty} \hat{f}(n) = 0$). The

Dirichlet’s Theorem on point-wise convergence of Fourier series (If $f$ is Lebesgue integrable periodic function that is differentiable at a point $x_0$, then the Fourier series of $f$ at $x_0$ converges to $f(x_0)$) and convergence of the Fourier series of functions such as $f(x) = |x|$ on $[-\pi, \pi]$.


**Unit III. Fejer’s Theorem and applications** (15 Lectures)

Fejer’s Kernel, Fejer’s Theorem for a continuous $2\pi$-periodic function, density of trigonometric polynomials in $L^2(-\pi, \pi)$, Parseval’s identity.

Convergence of Fourier series of an $L^2$-periodic function w.r.t the $L^2$-norm, Riesz-Fisher theorem on Unitary isomorphism from $L^2(-\pi, \pi)$ onto the sequence space $l^2$ of square summable complex sequences.


**Unit IV. Dirichlet Problem in the unit disc** (15 Lectures)

Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The Poisson kernel, Abel summability, Abel summability of periodic continuous functions, Weierstrass Approximation Theorem as application, Solution of Dirichlet problem for the disc.

Applications of Fourier series to Isoperimetric inequality in the plane and Heat equation on the circle.


**PSMT403,PAMT403 Calculus on Manifolds**

**Unit I. Multilinear Algebra** (15 Lectures)

Multilinear map on a finite dimensional vector space $V$ over $\mathbb{R}$, and $k$-tensors on $V$, the collection $T^k(V)$ (or $\otimes^k(V^*)$) of all $k$-tensors on $V$, tensor product $S \otimes T$ of $S \in T^k(V)$ & $T \in T^k(V)$, Alternating tensors and the collection $\wedge^k V^*$ of all $k$-tensors on $V$, The exterior product (or wedge product, basis of $\wedge^k V^*$, orientations of a finite dimensional vector space $V$ over $\mathbb{R}$.)
Reference: pp 75-84 in Chapter 4 of M. Spivak, *Calculus on Manifolds*, W.A. Benjamin.

**Unit II. Differential Forms (15 Lectures)**

Differential forms: $k$-forms on $\mathbb{R}^n$, wedge product $\omega \wedge \eta$ of a $k$-form $\omega$ and $l$-form $\eta$, the exterior derivative and properties, Pull back of forms and properties, Closed and exact forms, Poincare's lemma.


**Unit III. Basics of submanifolds of $\mathbb{R}^n$ (15 Lectures)**

Submanifolds of $\mathbb{R}^n$, submanifolds of $\mathbb{R}^n$ with boundary, Smooth functions defined on Submanifolds of $\mathbb{R}^n$, Tangent vectors and Tangent spaces of Submanifolds of $\mathbb{R}^n$.

$p$-forms and differentiable $p$-forms on a submanifold of $\mathbb{R}^n$, exterior derivative $d\omega$ of any differentiable $p$-form on a submanifolds of $\mathbb{R}^n$, Orientable submanifolds of $\mathbb{R}^n$ and Oriented submanifolds of $\mathbb{R}^n$, Orientation preserving maps, Vector fields on submanifolds of $\mathbb{R}^n$, outward unit normal on the boundary of a submanifold of $\mathbb{R}^n$ with non-empty boundary, induced orientation of the boundary of an oriented submanifold of $\mathbb{R}^n$ with non-empty boundary.


**Unit IV. Stokes’s Theorem (15 Lectures)**

Integral $\int_{[0,1]^k} \omega$ of a $k$-form on the cube $[0,1]^k$, Integral $\int_c \omega$ of a $k$-form on an open subset $A$ of $\mathbb{R}^k$ where $c$ is a singular $k$-cube in $A$. Theorem (Stokes’ Theorem for $k$-cubes): If $\omega$ is a $(k-1)$-form on an open subset $A$ of $\mathbb{R}^k$ and $c$ is a singular $k$-cube in $A$, then $\int_c d\omega = \int \partial \omega$.

(Reference: pp 100-108 in Chapter 4 of M. Spivak, *Calculus on Manifolds*, W.A. Benjamin.)

Integration of a differentiable $k$-form on an oriented $k$-dimensional submanifold $M$ of $\mathbb{R}^n$: Change of variables theorem: If $c_1, c_2 : [0,1]^k \rightarrow M$ are two Orientation preserving maps in $M$ and $\omega$ is any $k$-form on $M$ such that $\omega = 0$ outside of $c_1([0,1]^k) \cap c_2([0,1]^k)$, then $\int_{c_1} \omega = \int_{c_2} \omega$, Stokes’ theorem for submanifolds of $\mathbb{R}^k$, Volume element, Integration of functions on a submanifold of $\mathbb{R}^k$, Classical theorems: Green’s theorem, Divergence theorem of Gauss, Green’s identities. (Reference: pp 122-137 in Chapter 4 of M. Spivak, *Calculus on Manifolds*, W.A. Benjamin Inc.)

**Additional Reference Books:**

Optional Course

This course has two parts: First part is the Optional Course I & the second part is Optional Course II.

Optional Course I: Linear Programming

Unit I. Linear Programming (15 Lectures)

Operations research and its scope, Necessity of operations research in industry, Linear programming problems, Convex sets, Simplex method, Theory of simplex method, Duality theory and sensitivity analysis, Dual simplex method.

Unit II. Transportation Problems (15 Lectures)
Transportation and Assignment problems of linear programming, Sequencing theory and Travelling salesman problem.

Optional Course II: Optimisation

Unit III. Unconstrained Optimization (15 Lectures)

First and second order conditions for local optima, One-Dimensional Search Methods: Golden Section Search, Fibonacci Search, Newtons Method, Secant Method, Gradient Methods, Steepest Descent Methods.

Unit IV. Constrained Optimization Problems (15 Lectures)

Problems with equality constraints, Tangent and normal spaces, Lagrange Multiplier Theorem, Second order conditions for equality constraints problems, Problems with inequality constraints, Karush-Kuhn-Tucker Theorem, Second order necessary conditions for inequality constraint problems.

Recommended Text Books:

Scheme of Examination

I. Semester End Theory Examinations:
The scheme of examination for the revised course in the subject of Mathematics for Semesters III & IV at the M.A./M.Sc. Programme (CBCS) will be as follows.

There shall be a Semester-end external Theory examination of 100 marks for all the courses of Semester III and IV- except for the project courses USMT405/UAMT405-to be conducted by the University.

Theory Question paper pattern:

a) There shall be five questions each of 20 marks.

b) On each unit there will be one question and the fifth one will be based on entire syllabus.

c) All questions shall be compulsory with internal choice within each question.

d) Each question may be subdivided into sub-questions a, b, c, and the allocations of marks depend on the weightage of the topic.

e) Each question will be of 30 marks when marks of all the subquestions are added (including the options) in that question.

II. Evaluation of Project work
(courses: USMT405/UAMT405):

The evaluation of the Project submitted by a student shall be made by a Committee appointed by the Head of the Department of Mathematics of the respective college.

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the respective college. This committee shall have two members, possibly with one external referee. Each project output shall be displayed on the website of the University.

The Marks for the project are detailed below:
### Contents of the project | 40 marks
## Presentation of the project | 30 marks
### Viva of the project | 30 marks.

Total Marks= 100 per project per student.

### III. Evaluation of Skill Course

At the end of Semester III, there shall be a Semester end Internal Examination of 100 marks for the evaluation of the Skill Course and students shall be given grades A, B, C, D (A being the highest grade and D being the lowest grade). A student shall be required to get minimum of C grade to qualify for the M.A./M.Sc. degree CBCS Programme in the subject of Mathematics. The marks of the Skill Course shall not be considered for the calculation of CGPA score of the M.A./M.Sc. degree.

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